TMUA TOPIC TEST

Transformations Part b Topic Test Solutions

Time limit: 40 minutes

Calculators: not permitted

Format: 10 multiple-choice questions

This is a TMUA-style topic test: a timed set of TMUA-style questions, with a particular focus on a given topic. You can also sit this test online at jzmaths.com, where the result will count towards your mastery score.

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Question 1

Let $f(t) = \sin t$ and define $g(x) = 2f(2\pi - |3x|) + 1$. How many solutions does $g(x) = 0$ have in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$?

A 6

B 3

C 4

D 8

Solution 1

Answer: C

The equation $g(x) = 0$ requires $f(2\pi - |3x|) = -1/2$. The function $f(2\pi - |3x|)$ is even, so we only need to count the positive solutions and double that count.

For $0 < x \le \pi/2$, the equation becomes

$$\sin(2\pi - 3x) = -\sin(3x) = -\frac{1}{2} \quad \Rightarrow \quad \sin(3x) = 1/2.$$

The function $\sin(3x)$ has period $2\pi/3$, and both solutions in one period occur in its first half, $0 < x < \pi/3$. Since $0 < x \le \pi/2$ lies within one complete period, there are exactly two positive solutions.

By symmetry, there are also two negative solutions. Hence there are $2 \times 2 = 4$ solutions.

Question 2

A point (p, q) on the graph of $y = f(x)$, where $q \neq 0$, produces exactly two corresponding points on the graph of $y = 5 - 2f(3 - |2x + 1|)$. The x-coordinates of these two points differ by 4, and their common y-coordinate is 9. What is the value of p/q?

A -2

B $\frac{1}{2}$

C 2

D $-\frac{1}{2}$

Solution 2

Answer: B

The new input to f must equal the old input p, so the two new x-coordinates satisfy

$$3 - |2x + 1| = p.$$

Since there are two distinct images, $3 - p > 0$, and the two solutions are

$$x = \frac{-1 \pm (3 - p)}{2}.$$

Their difference is $3 - p$, so $3 - p = 4$ and hence $p = -1$. The vertical transformation sends q to $5 - 2q$. Therefore

$$5 - 2q = 9 \quad \Rightarrow \quad q = -2.$$

It follows that $p/q = (-1)/(-2) = 1/2$.

Question 3

Let $k > 1$. Which of the following conditions both **guarantees** that the graph

$$y = |3 - 2\,|\ln(k - |2x - 1|)||$$

has exactly four distinct intersections with the x-axis and **is required** for this to occur?

A $k = e^{3/2}$

B $k \geq e^{3/2}$

C $k > e^2$

D $k > e^{3/2}$

E $1 < k \leq e^{3/2}$

Solution 3

Answer: D

I hope you can see that we are looking for a condition that is both sufficient and necessary for there to be four intersections!

Now, the curve is symmetric about $x = \frac{1}{2}$, so every intersection away from $x = \frac{1}{2}$ occurs as part of a symmetric pair. To have four intersections, we need two such pairs.

The central point $x = \frac{1}{2}$ lies on the x-axis when

$$|3 - 2|\ln k|| = 0,$$

giving $k = e^{-3/2}$ or $k = e^{3/2}$, but only $k = e^{3/2} > 1$ is valid.

Next, we should probably make a sketch to determine which side of $k = e^{3/2}$ gives four intersections. **But** look at the options!

Options A, B and E all include $k = e^{3/2}$, where there is a central intersection and hence an odd number of intersections, so none of them can guarantee exactly four. This leaves only C and D, both of which lie on the same side of $e^{3/2}$.

Option C, $k > e^2$, introduces an unnecessary additional restriction. A symmetric pair can appear or disappear only by merging at the centre, and the only critical value for $k > 1$ is $e^{3/2}$. Therefore, there is no further change at $k = e^2$: C is sufficient, but it is not needed.

Hence the condition that both guarantees four intersections and is required is $k > e^{3/2}$, which is option D. Laziness won! :)

Question 4

Given $f(x) = x^4 - 2x^2$, let the curve C be given by the 90° clockwise rotation about $(2, 5)$ of $y = |f(x)| + 1$. How many intercepts does C have with the line $x = -1$?

A 1

B 2

C 3

D 4

E 5

F 6

Solution 4

Answer: D

I can't be bothered to write an orthodox solution for this one, so I am going to come clean: here is how you **cheat** on this question...

Instead of rotating $y = |f(x)| + 1$, rotate the line $x = -1$ by 90° anticlockwise about $(2, 5)$, which is far easier and will equally tell you the number of intersections! :D The rotated line is $y = 2$. Now **do a rough sketch of the curve and the rotated line**; it becomes clear that you need to find a couple of turning points.

Differentiating f and equating to zero, you will find that the relevant turning points occur at $x = 1$ and, by symmetry, $x = -1$. Their y-coordinates are both -1, so their transformed y-coordinates are both 2. Therefore, there are 4 intersection points!

Question 5

The equation $f(u) = -2$ has exactly three real solutions, namely $u = -1$, $u = 2$ and $u = 5$, while the equation $f(u) = \frac{2}{3}$ has no real solutions. Define $g(x) = 4 - |3f(5 - |2x - 1|) + 2|$. How many real solutions does $g(x) = 0$ have?

A 3

B 4

C 5

D 6

Solution 5

Answer: C

Write $H(x) = 5 - |2x - 1|$. The equation $g(x) = 0$ is equivalent to

$$|3f(H(x)) + 2| = 4,$$

so

$$f(H(x)) = -2 \quad \text{or} \quad f(H(x)) = \frac{2}{3}.$$

The second equation gives no solutions. For the first, $H(x)$ must be one of -1, 2 and 5. For a value p, the equation $H(x) = p$ becomes

$$|2x - 1| = 5 - p.$$

For $p = -1$ and $p = 2$, the right-hand side is positive, giving two solutions each. For $p = 5$, it is zero, giving one solution. The total is therefore $2 + 2 + 1 = 5$.

Question 6

Let $f(t) = t^3 - 3t$ and define $g(x) = |f(2x-1) - 1| - 1$. How many distinct x-intercepts does the graph of $y = g(x)$ have?

A 3

B 4

C 6

D 5

Solution 6

Answer: D

Method 1: This is the preferred quick and dirty TMUA approach!

Just track the y-coordinate of stationary points, and make a rough sketch. :)

We have $f'(t) = 3t^2 - 3$, so the stationary values occur when $t = \pm 1$. The local maximum value is $f(-1) = 2$. Since f is odd, the local minimum value is therefore -2.

Under the vertical transformation $y \mapsto |y-1| - 1$, these two stationary values become

$$2 \mapsto 0 \qquad \text{and} \qquad -2 \mapsto 2.$$

Now sketch $g(x)$, and you can completely ignore the horizontal transformations as they do not affect the number of x-intercepts. Now with these known y-coordinates, you will immediately see there are 5 intersections!

Method 2: This is the orthodox boring approach...but I am going to be good and include it.

Put $u = 2x - 1$. Since this is a one-to-one change of variable, it does not change the number of distinct solutions. An x-intercept satisfies

$$|f(u) - 1| = 1,$$

which is equivalent to $f(u) = 0$ or $f(u) = 2$. These equations factor as

$$\begin{aligned} f(u) = 0 \quad &\iff \quad u(u^2 - 3) = 0, \\ f(u) = 2 \quad &\iff \quad u^3 - 3u - 2 = (u-2)(u+1)^2 = 0. \end{aligned}$$

The first equation has three distinct real roots. The second has two distinct real roots; the repeated factor contributes only one distinct root. The two sets cannot overlap because they give different values of $f(u)$. Hence there are $3 + 2 = 5$ distinct intercepts.

Question 7

Let $f_k(x) = x^4 - 4x^2 + k$. Starting with $y = f_k(x)$, Sequence P translates the graph horizontally by $+3$ units, translates it vertically by $+1$ unit, applies a vertical modulus transformation, and then reflects it in $y = 2$. Sequence Q applies the vertical modulus transformation first, then translates horizontally by $+3$ units, translates vertically by $+1$ unit, and reflects in $y = 2$. For which values of k do the two sequences produce the same final graph?

A $k \geq 4$

B $k > 4$

C $k \leq 4$

D $k \in \mathbb{R}$

Solution 7

Answer: A

Write $t = f_k(x - 3)$. Sequence P produces $y = 4 - |t + 1|$, while Sequence Q produces $y = 3 - |t|$. These outputs are equal exactly when

$$|t + 1| = |t| + 1.$$

For real t, this equality holds exactly when $t \geq 0$. The two graphs are therefore identical precisely when $f_k(x) \geq 0$ for every real x. Completing the square in x^2 gives

$$f_k(x) = (x^2 - 2)^2 + k - 4.$$

Its minimum value is $k - 4$, attained when $x^2 = 2$. Hence the required condition is $k - 4 \geq 0$, or $k \geq 4$.

Question 8

Let f be any real-valued function on $\mathbb{R}$, and define

$$g(x) = f(2-|x|)\big(2-f(2-|x|)\big), \qquad h(x) = f(2-x)\big(2-f(2-x)\big).$$

Which condition is **sufficient but not necessary** for $g(x) = h(x)$ for every real x?

A For every real t, $f(t) \geq -1$.

B For every real t, $f(2+t) = f(2-t)$.

C For every real t, $f(-t) = -f(t)$.

D The function f is one-to-one.

E For every real t, $f(2+t) = -f(2-t)$.

Solution 8

Answer: B

Define

$$T(y) = y(2-y) = 1-(y-1)^2.$$

Then $g(x) = T(f(2-|x|))$ and $h(x) = T(f(2-x))$.

For $x \geq 0$, we have $|x| = x$, so $g(x) = h(x)$ automatically.

For $x < 0$, we have $|x| = -x$ and hence $2-|x| = 2+x$. Condition B gives

$$f(2-|x|) = f(2+x) = f(2-x),$$

so applying T to both sides gives $g(x) = h(x)$. Therefore B is sufficient.

However, B is not necessary. Take $f(u) = u-1$, which is not symmetric about the input 2, since $f(2+t) = 1+t$ and $f(2-t) = 1-t$. Nevertheless,

$$g(x) = (1-|x|)(1+|x|) = 1-x^2$$

and

$$h(x) = (1-x)(1+x) = 1-x^2.$$

Thus $g(x) = h(x)$ for every real x, even though B does not hold. Therefore B is a correct option, so it must be the correct option unless I mistakenly put two correct options in the question!

For completeness, we will show the others do not work.

For A, take $f(u) = u^2$. This satisfies $f(t) \geq -1$ for every real t, but

$$g(-1) = 1, \qquad h(-1) = -63.$$

Thus A is not sufficient.

For C and D, take $f(u) = u$. This function is both odd and one-to-one, but

$$g(-1) = 1, \qquad h(-1) = -3.$$

Thus neither C nor D is sufficient.

Finally, for E, take $f(u) = u - 2$. For every real t,

$$f(2 + t) = t = -f(2 - t),$$

so E holds. However,

$$g(-1) = -3, \qquad h(-1) = 1,$$

so E is not sufficient.

Question 9

Let $f(x) = x^3 - 3x$. In both sequences, first stretch $y = f(x)$ horizontally by scale factor $1/2$ and then translate it horizontally by $+1$. Sequence P next translates vertically by $+2$ and then applies a vertical modulus transformation. Sequence Q applies the vertical modulus transformation before translating vertically by $+2$. Which statement is correct?

A Sequence P produces exactly one more local maximum than Sequence Q.

B Sequence Q produces exactly one more local maximum than Sequence P.

C Both sequences produce exactly one local maximum.

D Both sequences produce exactly two local maxima.

Solution 9

Answer: B

The horizontal transformations replace x by $2x - 2$, which does not change the number of local maxima, so it is just distraction noise that can be ignored! It is therefore enough to compare $P = |f(t) + 2|$ with $Q = |f(t)| + 2$.

Differentiating f, we find that its turning points are $(-1, 2)$ and $(1, -2)$. Therefore, when P is first translated up by $+2$, its local minimum is no longer negative, so the subsequent modulus transformation does not change it into a maximum. Hence P does not gain another local maximum compared with f.

However, for Q, the vertical modulus is applied immediately, so the negative local minimum at $(1, -2)$ is reflected in the horizontal axis and becomes a local maximum. The subsequent vertical translation by $+2$ does not change this.

Therefore, Q has one more local maximum than P.

Question 10

Let f be a function for which

$$\int_0^4 |f(u)-1|\,du = 3.$$

Define $g(x) = 5 - 2|f(4-|2x-1|)-1|$. What is the value of $\int_{-3/2}^{5/2} g(x)\,dx$?

A 8

B 14

C 17

D 20

Solution 10

Answer: B

The input $4-|2x-1|$ runs from 0 to 4 on $-3/2 \le x \le 1/2$, and then from 4 back to 0 on $1/2 \le x \le 5/2$. Let

$$J = \int_{-3/2}^{5/2} |f(4-|2x-1|)-1|\,dx.$$

Using $u = 3+2x$ on the first interval and $u = 5-2x$ on the second gives

$$J = \frac{1}{2}\int_0^4 |f(u)-1|\,du + \frac{1}{2}\int_0^4 |f(u)-1|\,du = 3.$$

In case you are wondering where the $\frac{1}{2}$s came from, the boring explanation is standard A-level integration by substitution. The more fashionable explanation is that $u = 2x+3$ and $u = 5-2x$ both involve a horizontal stretch with scale factor $\frac{1}{2}$, so the areas must also be multiplied by $\frac{1}{2}$.

Here is a good unrelated example to illustrate this. Compare the sizes of

$$\int_0^{\pi} \sin x\,dx \qquad \text{and} \qquad \int_0^{\pi/2} \sin(2x)\,dx.$$

In both integrals, the argument of sin ranges from 0 to π. Thinking graphically, the first integral is clearly twice the second, because the second graph has undergone a horizontal stretch with scale factor $\frac{1}{2}$, despite the argument of sin ranging over the same interval. I should probably have written this in an article somewhere... never mind. :)

Back to the question! Therefore,

$$\int_{-3/2}^{5/2} g(x)\,dx = \int_{-3/2}^{5/2} 5\,dx - 2J = 20 - 6 = 14.$$