TMUA TOPIC TEST

Transformations Part A Topic Test Solutions

Time limit: 60 minutes

Calculators: not permitted

Format: 10 multiple-choice questions

This is a TMUA-style topic test: a timed set of TMUA-style questions, with a particular focus on a given topic. You can also sit this test online at jzmaths.com, where the result will count towards your mastery score.

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Question 1

Let $f(t) = t^4 - 2t^3 + P(t)$, where P is a polynomial of degree at most 2. Define $g(x) = 3 - 2f(2-x)$. What is the coefficient of x^3 in $g(x)$?

A 6

B -6

C 12

D -12

Solution 1

Answer: C

The term $P(2-x)$ has degree at most 2, so it cannot contribute to the coefficient of x^3. In $f(2-x)$, the relevant coefficient is

$$\binom{4}{3}2(-1)^3 - 2(-1)^3 = -8 + 2 = -6.$$

The outer transformation $g(x) = 3 - 2f(2-x)$ multiplies this coefficient by -2, while the added constant 3 has no effect. Therefore, the required coefficient is

$$-2 \times (-6) = 12.$$

Question 2

Starting with an arbitrary curve, consider the following two sequences. Sequence P reflects the curve in $x = 3$, stretches it horizontally by scale factor $1/2$, reflects it in $y = -2$, and then stretches it vertically by scale factor 3. Sequence Q stretches the curve horizontally by scale factor $1/2$, reflects it in $x = a$, stretches it vertically by scale factor 3, and then reflects it in $y = b$. For which pair (a, b) do the two sequences always produce the same final curve?

A $\quad (a, b) = \left(6, -\dfrac{3}{2}\right)$

B $\quad (a, b) = \left(\dfrac{3}{2}, 6\right)$

C $\quad (a, b) = \left(-\dfrac{3}{2}, -6\right)$

D $\quad (a, b) = \left(\dfrac{3}{2}, -6\right)$

Solution 2

Answer: D

Track an arbitrary point (p, q) rather than trying to write an equation for the whole curve. Under Sequence P, the point moves as follows:

$$(p, q) \mapsto (6 - p, q) \mapsto \left(3 - \frac{p}{2}, q\right) \mapsto \left(3 - \frac{p}{2}, -4 - q\right) \mapsto \left(3 - \frac{p}{2}, -12 - 3q\right).$$

Under Sequence Q, its final image is

$$(p, q) \mapsto \left(\frac{p}{2}, q\right) \mapsto \left(2a - \frac{p}{2}, q\right) \mapsto \left(2a - \frac{p}{2}, 3q\right) \mapsto \left(2a - \frac{p}{2}, 2b - 3q\right).$$

For these images to agree for every p and q, their constant parts must agree. Hence

$$2a = 3, \qquad 2b = -12,$$

so $(a, b) = (3/2, -6)$.

Question 3

Starting with the graph of $y = f(x)$, each option describes a sequence of transformations. Treat f as an arbitrary function with no special symmetry or periodicity. Which option produces a resulting function that is different from the functions produced by every one of the other five options?

A Translate horizontally by $+2$ units; stretch horizontally by scale factor $\frac{1}{3}$; stretch vertically by scale factor 2; reflect in the line $y = \frac{5}{2}$.

B Reflect in the x-axis; stretch vertically by scale factor 2; translate vertically by $+5$ units; stretch horizontally by scale factor $\frac{1}{3}$; translate horizontally by $+\frac{2}{3}$ units.

C Stretch horizontally by scale factor $\frac{1}{3}$; translate horizontally by $+\frac{2}{3}$ units; reflect in the x-axis; stretch vertically by scale factor 2; translate vertically by $+5$ units.

D Translate horizontally by -2 units; stretch horizontally by scale factor $\frac{1}{3}$; stretch vertically by scale factor 2; reflect in the line $y = \frac{5}{2}$.

E Stretch horizontally by scale factor $\frac{1}{3}$; translate horizontally by $-\frac{2}{3}$ units; reflect in the x-axis; stretch vertically by scale factor 2; translate vertically by $+5$ units.

F Translate horizontally by $+2$ units; stretch horizontally by scale factor $\frac{1}{3}$; reflect in the line $y = \frac{5}{2}$; stretch vertically by scale factor 2.

Solution 3

Answer: F

A horizontal transformation replaces x in the whole current function, while a vertical transformation acts on the whole current output. Applying those rules gives the following three groups of final functions:

$$\begin{aligned} \text{A, B and C:} \quad & y = 5 - 2f(3x - 2), \\ \text{D and E:} \quad & y = 5 - 2f(3x + 2), \\ \text{F:} \quad & y = 10 - 2f(3x - 2). \end{aligned}$$

Thus F alone produces a function different from all the others.

Question 4

The circle $x^2 + y^2 = 1$ is first stretched horizontally by scale factor 2, then reflected in the line $x = 3$, then stretched vertically by scale factor 3, and finally reflected in the line $y = 1$. Which equation represents the resulting curve?

A $\quad \dfrac{(x-6)^2}{4} + \dfrac{(y-2)^2}{9} = 1$

B $\quad \dfrac{(x-3)^2}{4} + \dfrac{(y-1)^2}{9} = 1$

C $\quad \dfrac{(x-6)^2}{9} + \dfrac{(y-2)^2}{4} = 1$

D $\quad \dfrac{(x+6)^2}{4} + \dfrac{(y+2)^2}{9} = 1$

Solution 4

Answer: A

We take a first-principles approach by tracking the image of an arbitrary point (p, q) on the original curve. We then express p and q in terms of the coordinates of its image and impose the original condition $p^2 + q^2 = 1$.

Track (p, q) through the four transformations:

$$(p, q) \mapsto (2p, q) \mapsto (6 - 2p, q) \mapsto (6 - 2p, 3q) \mapsto (6 - 2p, 2 - 3q) = (X, Y).$$

Therefore,

$$X = 6 - 2p, \qquad Y = 2 - 3q,$$

so

$$p = \frac{6 - X}{2}, \qquad q = \frac{2 - Y}{3}.$$

Since $p^2 + q^2 = 1$, we obtain

$$\left(\frac{6 - X}{2}\right)^2 + \left(\frac{2 - Y}{3}\right)^2 = 1.$$

Hence the equation of the image curve is

$$\frac{(X - 6)^2}{4} + \frac{(Y - 2)^2}{9} = 1.$$

Replacing X and Y with the usual coordinate variables x and y, the resulting equation is

$$\frac{(x-6)^2}{4} + \frac{(y-2)^2}{9} = 1.$$

Remark: Due to the different x and y scaling, tracking centre and radius won't work very well, this is where the first principle approach shines.

Question 5

Let f be a function whose derivative exists for all real x, with $f(2) = 3$ and $f'(2) = -1$. Starting with $y = f(x)$, reflect the graph in the line $x = 1$, stretch it horizontally by scale factor 2, reflect it in the line $y = 4$, and then stretch it vertically by scale factor $1/2$. What is the equation of the tangent to the resulting graph at the image of $(2, 3)$?

A $y = -\frac{1}{4}x + \frac{5}{2}$

B $y = \frac{1}{4}x + \frac{5}{2}$

C $y = -4x + \frac{5}{2}$

D $y = -\frac{1}{4}x + \frac{3}{2}$

Solution 5

Answer: A

Following the transformations in order, the resulting function is

$$g(x) = \frac{1}{2}\left(8 - f\left(2 - \frac{x}{2}\right)\right) = 4 - \frac{1}{2}f\left(2 - \frac{x}{2}\right).$$

The old input 2 occurs when $2 - x/2 = 2$, so the image point has $x = 0$. Its y-coordinate is $g(0) = 4 - f(2)/2 = 5/2$. Differentiating gives

$$g'(x) = \frac{1}{4}f'\left(2 - \frac{x}{2}\right),$$

so $g'(0) = f'(2)/4 = -1/4$. Therefore, the tangent is

$$y - \frac{5}{2} = -\frac{1}{4}x,$$

which is $y = -x/4 + 5/2$.

Question 6

Let $f(x) = \log_4 x$. Sequence P translates the graph horizontally by $+1$, stretches it horizontally by scale factor 2, translates it vertically by $+1$, and then stretches it vertically by scale factor 2, producing $y = g(x)$. Sequence Q stretches the graph horizontally by scale factor 2, translates it horizontally by $+1$, stretches it vertically by scale factor 2, and then translates it vertically by $+1$, producing $y = h(x)$. What is the set of x-coordinates of the points at which the two final graphs intersect?

A $\emptyset$

B $\left\{\frac{7}{3}\right\}$

C $\{3\}$

D $\{2\}$

Solution 6

Answer: C

Following the two orders carefully gives

$$g(x) = 2\log_4\left(\frac{x-2}{2}\right) + 2, \qquad h(x) = 2\log_4\left(\frac{x-1}{2}\right) + 1.$$

The common domain is $x > 2$. Equating the functions and simplifying gives

$$\log_4\left(\frac{x-2}{x-1}\right) = -\frac{1}{2}.$$

Therefore

$$\frac{x-2}{x-1} = 4^{-1/2} = \frac{1}{2},$$

which gives $x = 3$. This lies in the common domain, so the required set is $\{3\}$.

Question 7

Starting with $y = \sin x$, Sequence P translates the graph horizontally by $+\pi/3$, stretches it horizontally by scale factor $1/2$, reflects it in the y-axis, and translates it vertically by $+1$. Sequence Q reflects the graph in the x-axis, stretches it horizontally by scale factor $1/2$, translates it horizontally by $+a$, and translates it vertically by $+1$. For which value of a in the interval $0 \leq a < \pi$ do the two sequences produce the same final graph?

A $\frac{\pi}{6}$

B $\frac{\pi}{3}$

C $\frac{2\pi}{3}$

D $\frac{5\pi}{6}$

Solution 7

Answer: D

Following Sequence P in the stated order gives

$$g(x) = 1 + \sin\left(-2x - \frac{\pi}{3}\right) = 1 - \sin\left(2x + \frac{\pi}{3}\right).$$

Sequence Q gives

$$h(x) = 1 - \sin(2(x - a)) = 1 - \sin(2x - 2a).$$

For these functions to be equal for every x, the two sine functions must have phase shifts differing by a whole number of periods. Since the period of sine is 2π, we require

$$\left(2x + \frac{\pi}{3}\right) - (2x - 2a) = 2k\pi$$

for some integer k. Hence

$$\frac{\pi}{3} + 2a = 2k\pi,$$

so

$$a = k\pi - \frac{\pi}{6}.$$

The only value satisfying $0 \leq a < \pi$ is obtained when $k = 1$, giving

$$a = \frac{5\pi}{6}.$$

Question 8

Let f have period 3 and satisfy $\int_0^3 f(x)\,dx = 5$. Sequence P translates $y = f(x)$ horizontally by $+3$, stretches it horizontally by scale factor $1/2$, translates it vertically by $+2$, and stretches it vertically by scale factor 3, producing $y = g(x)$. Sequence Q performs each horizontal pair and each vertical pair in the reverse order, producing $y = h(x)$. What is

$$\frac{\int_0^{3/2} g(x)\,dx}{\int_0^{3/2} h(x)\,dx}?$$

A 1

B $\dfrac{7}{11}$

C $\dfrac{3}{2}$

D $\dfrac{11}{7}$

Solution 8

Answer: D

Following the stated orders carefully gives

$$g(x) = 3f(2x-3) + 6, \qquad h(x) = 3f(2x-6) + 2.$$

Since f has period 3, the two occurrences of f are equal. Also, using $u = 2x - 3$ and periodicity,

$$\int_0^{3/2} f(2x-3)\,dx = \frac{1}{2}\int_{-3}^{0} f(u)\,du = \frac{5}{2}.$$

The same value is obtained with $f(2x-6)$ since f has period 3. Therefore

$$\int_0^{3/2} g(x)\,dx = 3 \times \frac{5}{2} + 6 \times \frac{3}{2} = \frac{33}{2},$$

while

$$\int_0^{3/2} h(x)\,dx = 3 \times \frac{5}{2} + 2 \times \frac{3}{2} = \frac{21}{2}.$$

The required ratio is $(33/2)/(21/2) = 11/7$.

Question 9

Starting with $y = f(x)$, where f has no special symmetry or periodicity, the following four transformations are each applied exactly once in a uniformly random order: translate horizontally by $+1$; stretch horizontally by scale factor $1/2$; translate vertically by $+3$; stretch vertically by scale factor 2. What is the probability that the final graph has equation $y = 2f(2x-1)+3$?

A $\frac{1}{6}$

B $\frac{1}{3}$

C $\frac{1}{4}$

D $\frac{1}{2}$

Solution 9

Answer: C

To obtain the input $2x-1$, the horizontal translation must occur before the horizontal stretch. Reversing those two would give $2x-2$. Similarly, to obtain the output $2f(\cdot)+3$, the vertical stretch must occur before the vertical translation; reversing those two would give $2f(\cdot)+6$.

The relative order of a horizontal transformation and a vertical transformation is irrelevant. Writing H_T and H_S for the horizontal translation and stretch, and V_T and V_S for the vertical translation and stretch, the target occurs exactly when

$$H_T \text{ precedes } H_S, \qquad V_S \text{ precedes } V_T.$$

Each condition has probability $1/2$, so the required probability is

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

Question 10

A transformation T acts on a function h by

$$T(h(x)) = 3 - 2h(2x - 1).$$

Starting with f, the same transformation is applied n times, where n is a positive integer. Which expression represents the resulting function $T^n f$?

A $1 + (-2)^n \left(f(1 + 2^n(x - 1)) - 1\right)$

B $1 + 2^n \left(f(1 + (-2)^n(x - 1)) - 1\right)$

C $3^n - 2^n f(2^n x - n)$

D $1 + (-2)^n \left(f(1 + 2^{-n}(x - 1)) - 1\right)$

Solution 10

Answer: A

Method 1

Separate the transformation into the input map $H(x) = 2x - 1$ and the output map $V(t) = 3 - 2t$. Both maps have fixed point 1, since $H(1) = 1$ and $V(1) = 1$.

To see why this fixed point is useful, write $x = 1 + d$, where $d = x - 1$ is the signed displacement of x from 1. Then

$$H(1 + d) = 2(1 + d) - 1 = 1 + 2d.$$

Thus, if a number starts with signed displacement d from 1, then H changes this displacement to $2d$. Repeated applications therefore give

$$1 + d \longmapsto 1 + 2d \longmapsto 1 + 4d \longmapsto \cdots \longmapsto 1 + 2^n d.$$

Since $d = x - 1$, it follows that

$$H^n(x) = 1 + 2^n(x - 1).$$

Similarly, write $t = 1 + e$, where $e = t - 1$ is the signed displacement of t from 1. Then

$$V(1 + e) = 3 - 2(1 + e) = 1 - 2e.$$

Thus, each application of V multiplies the signed displacement from 1 by -2:

$$1 + e \longmapsto 1 - 2e \longmapsto 1 + 4e \longmapsto 1 - 8e \longmapsto \cdots \longmapsto 1 + (-2)^n e.$$

Since $e = t - 1$, we have

$$V^n(t) = 1 + (-2)^n (t - 1).$$

Now $T(h(x)) = V(h(H(x)))$, so after n applications,

$$(T^n f)(x) = V^n \left(f(H^n(x)) \right).$$

Substituting the formulas for H^n and V^n gives

$$(T^n f)(x) = 1 + (-2)^n \left(f\left(1 + 2^n (x - 1)\right) - 1 \right).$$

Method 2

Alternatively, calculate $H(x)$, $H^2(x)$ and $H^3(x)$ explicitly, then use the emerging pattern to obtain a formula for $H^n(x)$. Do the same for $V(t)$, $V^2(t)$ and $V^3(t)$. The remainder of the solution then follows in the same way as in Method 1.