TMUA PRACTICE SET

Multivariate Factorisation Practice 1 Solutions

Time limit: none — work at your own pace

Calculators: not permitted

Format: 15 multiple-choice questions

This is a TMUA-style practice set, intended for familiarisation with a specific topic.

Last updated: 12 August 2026 at 10:42

Spotted an error? Please email jzmaths@hotmail.com.

Question 1

Factorise completely: $x^2 - 5xy + 6y^2$.

A $(x + 2y)(x - 3y)$

B $(x - 2y)(x - 3y)$

C $(x - 2y)(x + 3y)$

D $(x + y)(x - 6y)$

Solution 1

Answer: B

Notice that $x = 2y$ is a root, since substituting $x = 2y$ gives $4y^2 - 10y^2 + 6y^2 = 0$. Therefore, by the factor theorem, $(x - 2y)$ is a factor.

Dividing by $(x - 2y)$ gives $x^2 - 5xy + 6y^2 = (x - 2y)(x - 3y)$.

Hence the complete factorisation is $(x - 2y)(x - 3y)$.

Question 2

Factorise completely: $2a^2 + 5ab + 2b^2$.

A $(2a - b)(a - 2b)$

B $(2a + b)(a - 2b)$

C $(2a - b)(a + 2b)$

D $(2a + b)(a + 2b)$

Solution 2

Answer: D

Put $a = tb$. Then $2a^2 + 5ab + 2b^2 = b^2(2t^2 + 5t + 2) = b^2(2t + 1)(t + 2)$. Hence the complete factorisation is $(2a + b)(a + 2b)$.

Question 3

Factorise completely: $3m^2 - 7mn + 2n^2$.

A $(3m - n)(m - 2n)$

B $(3m + n)(m - 2n)$

C $(3m - n)(m + 2n)$

D $(3m - 2n)(m - n)$

Solution 3

Answer: A

Notice that $m = 2n$ is a root, since substituting $m = 2n$ gives $12n^2 - 14n^2 + 2n^2 = 0$. Therefore, by the factor theorem, $(m - 2n)$ is a factor.

Dividing by $(m - 2n)$ gives $3m^2 - 7mn + 2n^2 = (m - 2n)(3m - n)$.

Hence the complete factorisation is $(3m - n)(m - 2n)$.

Question 4

Factorise completely: $4p^2 - 4pq - 3q^2$.

A $(2p - q)(2p + 3q)$

B $(2p - q)(2p - 3q)$

C $(2p + q)(2p - 3q)$

D $(4p + 3q)(p - q)$

Solution 4

Answer: C

Put $p = tq$. Then $4p^2 - 4pq - 3q^2 = q^2(4t^2 - 4t - 3) = q^2(2t + 1)(2t - 3)$. Hence the complete factorisation is $(2p + q)(2p - 3q)$.

Question 5

Factorise completely over the real numbers: $x^3 - x^2y - 4y^3$.

A $(x+2y)(x^2 - xy + 2y^2)$

B $(x-2y)(x^2 - xy + 2y^2)$

C $(x-2y)(x^2 + xy + 2y^2)$

D $(x-2y)(x^2 + 2xy + 2y^2)$

Solution 5

Answer: C

Put $x = ty$. Then the expression becomes $y^3(t^3 - t^2 - 4)$. Since $t = 2$ makes $t^3 - t^2 - 4$ equal to 0, $(t-2)$ is a factor. Dividing gives $t^3 - t^2 - 4 = (t-2)(t^2 + t + 2)$. The quadratic $t^2 + t + 2$ has discriminant $1 - 8 = -7 < 0$, so it has no further real linear factors. Therefore the complete factorisation is $(x-2y)(x^2 + xy + 2y^2)$.

Question 6

Factorise completely over the real numbers: $a^3 + a^2b - ab^2 + 15b^3$.

A $(a+3b)(a^2 - 2ab + 5b^2)$

B $(a-3b)(a^2 + 2ab + 5b^2)$

C $(a+3b)(a^2 + 2ab + 5b^2)$

D $(a+3b)(a^2 - 2ab - 5b^2)$

Solution 6

Answer: A

Put $a = tb$. Then the expression becomes $b^3(t^3 + t^2 - t + 15)$. Since $t = -3$ makes this equal to 0, $(t+3)$ is a factor. Dividing gives $t^3 + t^2 - t + 15 = (t+3)(t^2 - 2t + 5)$. The quadratic $t^2 - 2t + 5$ has discriminant $4 - 20 = -16 < 0$, so it has no further real linear factors. Therefore the complete factorisation is $(a+3b)(a^2 - 2ab + 5b^2)$.

Question 7

Factorise completely: $2x^3 - x^2y - 7xy^2 + 6y^3$.

A $(x - y)(x + 2y)(2x - 3y)$

B $(x - y)(x - 2y)(2x + 3y)$

C $(x + y)(x + 2y)(2x - 3y)$

D $(x - y)(2x + y)(x - 3y)$

Solution 7

Answer: A

Notice that $x = y$ is a root, since substituting $x = y$ gives $2y^3 - y^3 - 7y^3 + 6y^3 = 0$. Therefore, by the factor theorem, $(x - y)$ is a factor.

Dividing by $(x - y)$ gives $2x^3 - x^2y - 7xy^2 + 6y^3 = (x - y)(2x^2 + xy - 6y^2)$.

Now notice that $x = -2y$ is a root of the quadratic factor, since $2(-2y)^2 + (-2y)y - 6y^2 = 0$. Therefore $(x + 2y)$ is a factor.

Hence $2x^2 + xy - 6y^2 = (x + 2y)(2x - 3y)$, so the complete factorisation is $(x - y)(x + 2y)(2x - 3y)$.

Question 8

Factorise completely: $2a^3 + 3a^2b - 3ab^2 - 2b^3$.

A $(a + b)(2a + b)(a + 2b)$

B $(a - b)(2a + b)(a + 2b)$

C $(a - b)(2a - b)(a + 2b)$

D $(a + b)(2a - b)(a - 2b)$

Solution 8

Answer: B

Notice that $a = b$ is a root, since substituting $a = b$ gives $2b^3 + 3b^3 - 3b^3 - 2b^3 = 0$. Therefore, by the factor theorem, $(a - b)$ is a factor.

Dividing by $(a - b)$ gives $2a^3 + 3a^2b - 3ab^2 - 2b^3 = (a - b)(2a^2 + 5ab + 2b^2)$.

Now notice that $a = -2b$ is a root of the quadratic factor, since $2(-2b)^2 + 5(-2b)b + 2b^2 = 0$. Therefore $(a + 2b)$ is a factor.

Hence $2a^2 + 5ab + 2b^2 = (a + 2b)(2a + b)$, so the complete factorisation is $(a - b)(2a + b)(a + 2b)$.

Question 9

Factorise completely: $p^3 + 2p^2q - 5pq^2 - 6q^3$.

A $(p+q)(p-2q)(p+3q)$

B $(p+q)(p+2q)(p-3q)$

C $(p-q)(p-2q)(p+3q)$

D $(p+q)(p-2q)(p-3q)$

Solution 9

Answer: A

Notice that $p = 2q$ is a root, since substituting $p = 2q$ gives $8q^3 + 8q^3 - 10q^3 - 6q^3 = 0$. Therefore, by the factor theorem, $(p-2q)$ is a factor.

Dividing by $(p-2q)$ gives $p^3 + 2p^2q - 5pq^2 - 6q^3 = (p-2q)(p^2 + 4pq + 3q^2)$.

Now notice that $p = -q$ is a root of the quadratic factor, so $(p+q)$ is a factor.

Hence $p^2 + 4pq + 3q^2 = (p+q)(p+3q)$, so the complete factorisation is $(p+q)(p-2q)(p+3q)$.

Question 10

Factorise completely: $x^2 + xy - 3x - 2y^2 + 3y$.

A $(x - y)(x + 2y - 3)$

B $(x + y)(x - 2y - 3)$

C $(x - y)(x + 2y + 3)$

D $(x + y)(x - 2y + 3)$

Solution 10

Answer: A

Notice that $x = y$ is a root, since substituting $x = y$ gives $y^2+y^2-3y-2y^2+3y = 0$. Therefore, by the factor theorem, $(x-y)$ is a factor. Dividing by $(x-y)$ gives $x^2+xy-3x-2y^2+3y = (x-y)(x+2y-3)$. Hence the complete factorisation is $(x - y)(x + 2y - 3)$.

Question 11

Factorise completely: $a^3 + a^2b + 3ab + 3b^2 - 4a - 4b$.

A $(a + b)(a^2 + 3b - 4)$

B $(a - b)(a^2 + 3b - 4)$

C $(a + b)(a^2 - 3b - 4)$

D $(a + b)(a^2 + 3b + 4)$

Solution 11

Answer: A

Notice that $a = -b$ is a root, since substituting $a = -b$ gives $-b^3 + b^3 - 3b^2 + 3b^2 + 4b - 4b = 0$. Therefore, by the factor theorem, $(a + b)$ is a factor. Dividing by $(a + b)$ gives $a^3 + a^2b + 3ab + 3b^2 - 4a - 4b = (a + b)(a^2 + 3b - 4)$. Hence the complete factorisation is $(a + b)(a^2 + 3b - 4)$.

Question 12

Factorise completely: $2m^2 - 5mn + 2n^2 + 3m - 6n$.

A $(m - 2n)(2m - n + 3)$

B $(m + 2n)(2m - n + 3)$

C $(m - 2n)(2m + n + 3)$

D $(m - 2n)(2m - n - 3)$

Solution 12

Answer: A

Notice that $m = 2n$ is a root, since substituting $m = 2n$ gives $8n^2 - 10n^2 + 2n^2 + 6n - 6n = 0$. Therefore, by the factor theorem, $(m - 2n)$ is a factor. Dividing by $(m - 2n)$ gives $2m^2 - 5mn + 2n^2 + 3m - 6n = (m - 2n)(2m - n + 3)$. Hence the complete factorisation is $(m - 2n)(2m - n + 3)$.

Question 13

Factorise completely: $p^2 + pq - 3p - 4q - 4$.

A $(p + 4)(p + q - 1)$

B $(p - 4)(p + q - 1)$

C $(p - 4)(p + q + 1)$

D $(p + 4)(p - q - 1)$

Solution 13

Answer: C

Treat the expression as a polynomial in p. The numerical value $p = 4$ makes the expression equal to 0 for every q, so $(p - 4)$ is a factor. Dividing gives $p + q + 1$. Hence the complete factorisation is $(p - 4)(p + q + 1)$.

Question 14

Factorise completely: $x^2 - 2xy + 4xz - 2yz + 3z^2$.

A $(x+z)(x-2y+3z)$

B $(x-z)(x-2y-3z)$

C $(x+z)(x+2y+3z)$

D $(x-z)(x+2y-3z)$

Solution 14

Answer: A

Treat the expression as a polynomial in x, with y and z fixed. Setting $x = -z$ gives $z^2 + 2yz - 4z^2 - 2yz + 3z^2 = 0$, so $(x+z)$ is a factor. Dividing gives $x - 2y + 3z$. Therefore the complete factorisation is $(x+z)(x-2y+3z)$.

Question 15

Factorise completely: $2a^2 + ab^2 - 2ab - ac - b^3 + bc$.

A $(a-b)(2a+b^2-c)$

B $(a+b)(2a+b^2-c)$

C $(a-b)(2a-b^2-c)$

D $(a-b)(2a+b^2+c)$

Solution 15

Answer: A

Notice that $a = b$ is a root, since substituting $a = b$ gives $2b^2 + b^3 - 2b^2 - bc - b^3 + bc = 0$. Therefore, by the factor theorem, $(a-b)$ is a factor. Dividing by $(a-b)$ gives $2a^2 + ab^2 - 2ab - ac - b^3 + bc = (a-b)(2a+b^2-c)$. Hence the complete factorisation is $(a-b)(2a+b^2-c)$.