JZ Mock Set E Paper 2

Recommended time: 2 hours

Calculators: not permitted

Format: 20 multiple-choice questions

This is a TMUA-style mock paper modelled on the Test of Mathematics for University Admission. The TMUA is used in admissions for mathematics, economics, computer science, and engineering courses at universities including Cambridge, Oxford, Imperial College London, UCL, LSE, Warwick, and Durham.

Paper Guidance

- These papers contain only hard, very hard, and occasionally *unreasonably hard* TMUA-style questions :). I have deliberately left out the routine and easier material, so please do not be discouraged if you find a paper difficult or need longer than the recommended time. That is exactly how it is meant to feel. Take your time, enjoy the challenge, and remember that struggling with difficult problems is one of the best ways to get better.
- As a rough and deliberately conservative guide, a student who scores above 27 out of 40 across one full set, meaning both papers in four hours, is very likely already working at around the 7.0+ level on the current TMUA scale. However, this is only intended as a guide, not something to worry about too much. The main purpose of these papers is improvement rather than measurement: meet difficult problems, struggle with them, learn from them, and focus on getting better each time.

Best of luck, and enjoy the challenge!

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Question 1

Let t be a real number, and define

$$f(x) = x^3 - 3tx + 1.$$

Consider the following statements.

P: f is increasing on the interval $[-1, 1]$.

Q: The equation $f(x) = 1$ has exactly one solution in the interval $[-1, 1]$.

A P is sufficient but not necessary for Q

B P is necessary but not sufficient for Q

C P is necessary and sufficient for Q

D P is neither sufficient nor necessary for Q

Question 2

Consider the following attempt to solve the equation $(x-2)\sqrt{x+3} = x^2 - 5x + 6$:

$$(x-2)\sqrt{x+3} = x^2 - 5x + 6 \quad \text{(I)}$$

$$(x-2)\sqrt{x+3} = (x-2)(x-3) \quad \text{(II)}$$

$$\sqrt{x+3} = x - 3 \quad \text{(III)}$$

$$x + 3 = (x-3)^2 \quad \text{(IV)}$$

$$x + 3 = x^2 - 6x + 9 \quad \text{(V)}$$

$$x^2 - 7x + 6 = 0 \quad \text{(VI)}$$

$$(x-1)(x-6) = 0 \quad \text{(VII)}$$

The solutions of the original equation are concluded to be $x = 1$ and $x = 6$.

Which one of the following is true?

A The conclusion is entirely correct.

B The conclusion is not entirely correct; the first error occurs in passing from (I) to (II).

C The conclusion is not entirely correct; the first error occurs in passing from (II) to (III).

D The conclusion is not entirely correct; the first error occurs in passing from (III) to (IV).

E The conclusion is not entirely correct; the first error occurs in passing from (IV) to (V).

F The conclusion is not entirely correct; the first error occurs in passing from (V) to (VI).

G The conclusion is not entirely correct; the first error occurs in passing from (VI) to (VII).

Question 3

A real number x is such that **exactly one** of the following six statements is true.

I $x^2 < 9$

II $|x-1| < 1$

III $x, 2, 4$ are the first three terms of a geometric sequence with positive common ratio

IV $\log_2(x+1)$ is defined and less than 1

V x is a positive solution of $u^2 - 3u + 2 = 0$

VI the circle $X^2 + Y^2 = 1$ meets the line $X = x$ in two distinct points

A I

B II

C III

D IV

E V

F VI

Question 4

Find the sum of all solutions x in the interval $0 \leq x < 2\pi$ to

$$|\sin x| + |\cos x| = 1.$$

A π

B $\frac{3\pi}{2}$

C 2π

D $\frac{5\pi}{2}$

E 3π

F $\frac{7\pi}{2}$

Question 5

The real numbers x and y are such that exactly two of the following five statements are true. Which two are true?

I $\quad x < y$

II $\quad x < |y|$

III $\quad |x| < |y|$

IV $\quad x \geq 0$

V $\quad x < 0$

A I and IV

B I and V

C II and IV

D II and V

E III and IV

F III and V

Question 6

Let $c > 0$ be a fixed real constant. For each real number b, define

$$f_b(x) = x^2 - 2bx + c,$$

and let P_b denote the vertex of the graph $y = f_b(x)$. As b varies over the real numbers, the point P_b traces out a curve Γ in the (x, y)-plane.

Consider the following three statements.

(I) The curve Γ has equation $y = c + x^2$.

(II) For all real numbers $b_1 < b_2$, the point P_{b_2} lies strictly to the right of, and strictly below, the point P_{b_1}.

(III) There is a unique real value of b for which f_b has a repeated root.

Which of the statements are necessarily true?

A (I) only

B (II) only

C (III) only

D (I) and (II) only

E (I) and (III) only

F (II) and (III) only

G all of them

H none of them

Question 7

Positive real numbers x, y and z are such that

$$x, y, z$$

are consecutive terms of an arithmetic sequence, while

$$\log_2 x, \log_2 y, \log_2 z$$

are consecutive terms of an arithmetic sequence.

It is also known that

$$x + y + z = 12.$$

Find xyz.

A 32

B 48

C 60

D 64

E 72

F 96

Question 8

Consider triangles ABC such that $AB = 1$, $\sin A = \frac{1}{3}$ and $\cos C = x$.

What is the greatest possible value of x for which there is **exactly one** distinct triangle satisfying these conditions?

A

$$\frac{1}{3}$$

B

$$\frac{\sqrt{2}}{3}$$

C

$$\frac{2}{3}$$

D

$$\frac{2\sqrt{2}}{3}$$

E

$$\frac{1+\sqrt{2}}{3}$$

Question 9

For real t, define

$$A(t) = \int_{-1}^{1} |x^2 - t| \, dx.$$

Find the minimum possible value of $A(t)$.

A $\frac{1}{4}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{2}{3}$

E $\frac{3}{4}$

F 1

Question 10

A polynomial $p(x)$ has degree at most 3. When $p(x)$ is divided by $x-1$, $x-2$ and $x-4$, the remainders are 1, 2 and 4 respectively.

It is also known that $p(0)=0$.

Find the coefficient of x^3 in $p(x)$.

A -3

B -2

C -1

D 0

E 1

F 2

G 3

Question 11

Let f be a real function and let a be a real number.

We say that f is **smoothly joined** at $x = a$ if and only if, for every real number $h > 0$, there exists a real number $d > 0$ such that, for every real number x, if $|x - a| < d$, then $|f(x) - f(a)| < h$.

Which of the following means that f is **not smoothly joined** at $x = a$?

A For every real number $h > 0$, for every real number $d > 0$, there exists a real number x such that $|x - a| < d$ and $|f(x) - f(a)| \geq h$.

B There exists a real number $h > 0$ such that there exists a real number $d > 0$ such that, for every real number x, if $|x - a| < d$, then $|f(x) - f(a)| \geq h$.

C There exists a real number $h > 0$ such that, for every real number $d > 0$, there exists a real number x such that $|x - a| < d$ and $|f(x) - f(a)| \geq h$.

D There exists a real number $h > 0$ such that, for every real number $d > 0$, for every real number x, if $|x - a| < d$, then $|f(x) - f(a)| \geq h$.

E For every real number $h > 0$, there exists a real number $d > 0$ such that, for every real number x, if $|x - a| < d$, then $|f(x) - f(a)| \geq h$.

Question 12

Let f be a function. Each of the following is a statement about all real numbers a (for instance, the first reads: for every a, if $f(x) > 0$ for all $x > a$, then $f(a) < 0$). **Exactly one** of them is true. Which one is it?

A $f(x) > 0$ for all $x > a$ only if $f(a) < 0$

B $f(a) < 0$ and $f(x) > 0$ for all $x > a$ cannot both hold

C $f(x) > 0$ for all $x > a$ is necessary for $f(a) < 0$

D if $f(a) \geq 0$ then $f(x) \leq 0$ for some $x > a$

E $f(x) > 0$ for all $x > a$ implies $f(a) \geq 0$

F $f(a) \geq 0$ if and only if $f(x) \leq 0$ for some $x > a$

G $f(a) < 0$ is sufficient for there to exist $x > a$ with $f(x) \leq 0$

Question 13

For some positive integer n, three consecutive coefficients in the expansion of $(1 + x)^n$ are in the ratio $1 : 3 : 5$. What is the value of n?

A 5

B 7

C 12

D 15

E 18

Question 14

The two diagonals of the convex quadrilateral Q have equal length. A quadrilateral is convex if each of its interior angles is less than $180°$.

Consider the following statements.

(I) The four midpoints of the sides of Q are the vertices of a rhombus.

(II) At least one pair of opposite sides of Q is parallel.

(III) Q has at least one line of symmetry.

(IV) Let A, B, C, D be the vertices of Q, labelled in order, then $AB^2 + CD^2 = BC^2 + DA^2$.

Which of these statements is/are **necessarily** true for the quadrilateral Q?

A (I) only

B (I) and (II) only

C (I) and (III) only

D (II) and (IV) only

E (II) and (III) only

F (I) and (IV) only

G (I), (III) and (IV) only

H (II), (III) and (IV) only

I all of them

J none of them

Question 15

For a real number x, let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. The fractional part of x is then defined by

$$\{x\} = x - \lfloor x \rfloor,$$

so $0 \leq \{x\} < 1$.

For example, $\lfloor 3.7 \rfloor = 3$, so $\{3.7\} = 3.7 - 3 = 0.7$.

Also, $\lfloor -1.4 \rfloor = -2$, since $-2 \leq -1.4 < -1$. Therefore $\{-1.4\} = -1.4 - (-2) = 0.6$.

Define

$$f(x) = \{\{2x\} + \{3x\}\}.$$

How many values of x with $0 \leq x < 1$ satisfy $f(f(x)) = 1 - x$?

A 20

B 21

C 24

D 25

E 26

F 30

G 31

Question 16

Let f be a positive function defined on $[0, \infty)$ and twice differentiable on $(0, \infty)$. Define

$$I = \int_0^4 f(x)\, dx,$$

and let J_n be the trapezium rule estimate of I using n strips of equal width, where n is a positive integer.

Which of the following statements are necessarily true?

(1) If $f''(x) > 0$ for all $0 < x < 4$, then $J_n > I$ for every positive integer n.

(2) If $f''(x) \geq 0$ and $f'(x) > 0$ for all $0 < x < 4$, then $J_{n+1} < J_n$ for every positive integer n.

(3) If $I < J_n$ for some positive integer n, then

$$\int_0^4 f(4-x)\, dx < J_{n+1}.$$

A (1) only.

B (2) only.

C (3) only.

D (1) and (2) only.

E (1) and (3) only.

F (2) and (3) only.

G none of them.

H all of them

Question 17

A sphere has volume 1 cubic unit. A regular tetrahedron is inscribed in the sphere, so that all four vertices of the tetrahedron lie on the sphere. A second sphere is then inscribed in this tetrahedron, so that it is tangent to all four faces of the tetrahedron.

This process is repeated indefinitely: inside each sphere, a regular tetrahedron is inscribed, and inside that tetrahedron another sphere is inscribed.

Find the sum of the volumes of all the spheres formed, including the original sphere.

A

$$\frac{5+\sqrt{5}}{4}$$

B

$$\frac{5}{4}$$

C

$$\frac{27}{26}$$

D

$$\frac{3+\sqrt{3}}{2}$$

E

$$\frac{9+3\sqrt{2}}{7}$$

F

$$\frac{4}{3}$$

Question 18

In a geometric progression, the third term is $3 - \sqrt{5}$ and the seventh term is $18 - 8\sqrt{5}$. Given that every term of the progression is positive, what is the sum to infinity?

A $\sqrt{5} - 1$

B 2

C $\sqrt{5} + 1$

D $3 - \sqrt{5}$

E $2\sqrt{5} + 4$

F $\dfrac{3 + \sqrt{5}}{2}$

G $3 + \sqrt{5}$

Question 19

In a previous JZ Maths mock paper, candidates were introduced to the following two question-setting policies.

Policy 1. A multiple-choice question involving five statements must be made as difficult as possible: a student must determine the truth value of all five statements in order to guarantee identifying the correct option. In particular, checking any four statements must not be sufficient.

Policy 2. Subject to Policy 1, the question must contain as few answer options as possible, so that it appears easier than it really is.

The disclaimer accompanying that question claimed that no such policies officially existed.

Alan Turing, a student sitting the next JZ Maths mock paper, has secretly learnt that JZ Maths does, in fact, follow both policies religiously. JZ Maths is, of course, completely unaware of this serious breach of question-setting security!

Alan is presented with a question containing five statements, labelled $(1), (2), (3), (4), (5)$. He does not know how many of the statements are true. The answer options are shown below, with each option specifying exactly which statements are true.

Without bothering to check the truth of any statement, Alan is able to identify the correct option.

Which option does he choose?

A Statements $(1), (3), (5)$ only.

B Statements $(3), (5)$ only.

C Statements $(1), (2), (3), (5)$ only.

D Statements $(1), (5)$ only.

E Statements $(1), (3), (4), (5)$ only.

F Statements $(1), (3)$ only.

Question 20

Warning: Students should be aware that JZ Maths policies may change at any time, entirely at the whim of the unstable chief mock examiner - Mr JZ.

Statement P: ABC is a right-angled triangle.

Which of the following statements are true?

(1) Statement P is true if and only if triangle ABC can be divided by a line segment into two smaller triangles, each of which is similar to triangle ABC.

(2) Statement P is necessary for triangle ABC to be divisible by a line segment into two smaller triangles whose areas, together with the area of triangle ABC, can be arranged to form a geometric sequence with common ratio not equal to 1.

(3) P is a necessary condition, that must be satisfied by all the triangles made when a convex kite is divided into four triangles by its diagonals.

(4) Statement P is sufficient for every SSA specification of triangle ABC to determine it uniquely up to congruence.

(5) Statement P is true if and only if four congruent copies of triangle ABC can be assembled edge-to-edge, without gaps or overlaps, so that their union is exactly a rhombus.

A Statements (1), (3), (5) only.

B Statements (3), (5) only.

C Statements (1), (2), (3), (5) only.

D Statements (1), (5) only.

E Statements (1), (3), (4), (5) only.

F Statements (1), (3) only.