JZ Mock Set E Paper 1

Recommended time: 2 hours

Calculators: not permitted

Format: 20 multiple-choice questions

This is a TMUA-style mock paper modelled on the Test of Mathematics for University Admission. The TMUA is used in admissions for mathematics, economics, computer science, and engineering courses at universities including Cambridge, Oxford, Imperial College London, UCL, LSE, Warwick, and Durham.

Paper Guidance

- These papers contain only hard, very hard, and occasionally *unreasonably hard* TMUA-style questions :). I have deliberately left out the routine and easier material, so please do not be discouraged if you find a paper difficult or need longer than the recommended time. That is exactly how it is meant to feel. Take your time, enjoy the challenge, and remember that struggling with difficult problems is one of the best ways to get better.
- As a rough and deliberately conservative guide, a student who scores above 27 out of 40 across one full set, meaning both papers in four hours, is very likely already working at around the 7.0+ level on the current TMUA scale. However, this is only intended as a guide, not something to worry about too much. The main purpose of these papers is improvement rather than measurement: meet difficult problems, struggle with them, learn from them, and focus on getting better each time.

Best of luck, and enjoy the challenge!

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Question 1

Find all real values of k for which

$$kx^2 + (k+3)x + (k+3) > 0$$

holds for every real number x.

A $k > 1$

B $k < -3$ or $k > 1$

C $-3 < k < 1$

D $k > 0$

E $k > -3$

F There are no such values of k

Question 2

For $0 < a < 2$, let $f(x) = |x - a|$. The trapezium rule with two equal strips is used to estimate

$$\int_0^2 f(x)\,dx.$$

Find the product of all values of a for which the trapezium estimate exceed the exact integral by $\frac{1}{4}$.

A $\dfrac{4}{3}$

B $\dfrac{3}{4}$

C 2

D 1

E $\dfrac{3}{2}$

F $\dfrac{2}{3}$

Question 3

The points $A = (0, 0)$ and $B = (6, 0)$ are given, and a point P lies on the line $y = x + 2$. Find the minimum possible value of $PA + PB$.

A $3\sqrt{10}$

B 9

C 8

D $2\sqrt{15}$

E $2\sqrt{17}$

Question 4

For how many integer values of c does the line $y = 2x + c$ meet the circle $x^2 + y^2 = 20$ at two distinct points?

A 19

B 18

C 17

D 16

E 21

F 24

Question 5

What is the value of

$$\int_{-2}^{3} \Big| |x| - 1 \Big| \, dx\,?$$

A 3

B $\dfrac{9}{2}$

C 4

D $\dfrac{7}{2}$

E $\dfrac{13}{4}$

F 5

Question 6

The real number x satisfies both

$$\frac{x+3}{x^2-4} \leq 0 \quad \text{and} \quad \frac{x^2-9}{x-1} > 0.$$

Find the total length of the set of possible values of x.

A 0

B 1

C 2

D 3

E 4

F 6

G 8

H ∞

Question 7

The curve C is defined by

$$x = y + \sqrt{y} + 1.$$

Find the equation of the tangent to C at the point where $x = 3$.

A $\quad y = \dfrac{2}{3}x - 1$

B $\quad y = \dfrac{3}{2}x - \dfrac{7}{2}$

C $\quad y = \dfrac{1}{2}x - \dfrac{1}{2}$

D $\quad y = \dfrac{3}{4}x - \dfrac{5}{4}$

E $\quad y = -\dfrac{2}{3}x + 3$

Question 8

How many square divisors greater than 1 does 810000 have?

A square divisor of an integer n is a factor of n which is also a square number.

A 24

B 26

C 27

D 30

E 15

F 18

Question 9

The sequence u_n is defined by $u_n = \frac{n}{2^{n-1}}$ for $n \geq 1$. For a positive integer n, let

$$S_n = \sum_{k=1}^{n} u_k.$$

Which of the following is an expression for S_n?

A

$$3 - \frac{4}{2^n}$$

B

$$2 + \frac{n-2}{2^{n-1}}$$

C

$$4 - \frac{n^2 - 2n + 4}{2^{n-1}}$$

D

$$4 - \frac{n+2}{2^{n-1}}$$

E

$$2 - \frac{n+2}{2^n}$$

F

$$4 - \frac{n}{2^{n-1}}$$

Question 10

For each real number a, let $F(a)$ be the area between the graph $y = |x - a|$ and the x-axis from $x = 0$ to $x = 2$. How many real values of a satisfy

$$F(a) = a + 1?$$

A 0

B 1

C 2

D 3

E 4

Question 11

Triangle ABC is right-angled at C, with

$$AC = 5, \qquad BC = 12, \qquad AB = 13.$$

The perpendicular from C to AB is drawn. Of the two smaller triangles formed, the larger triangle is selected.

Within the selected triangle, the perpendicular from its right-angled vertex to its hypotenuse is drawn, and the larger of the two new triangles is selected. This process is continued indefinitely.

Find the sum of the lengths of all the perpendiculars drawn.

A 60

B 30

C 48

D 80

E 72

F The sum does not converge.

Question 12

Find the sum of all positive values of k satisfying

$$\int_0^k (|2x-6|-2)\ dx = 3.$$

A 1

B 4

C 8

D 9

E 13

F 16

Question 13

The values of x, measured in degrees, satisfy both

$$\sin 3x + 2\cos 3x = \frac{\sqrt{3}+2}{2}$$

and

$$3\sin 3x - 2\cos 3x = \frac{3\sqrt{3}-2}{2}.$$

The sum of all such values of x in the interval $0 \leq x \leq k^\circ$ is 800°.

Find the least possible value of k.

A 240°

B 500°

C 380°

D 400°

E 360°

F 720°

Question 14

Real numbers a, b, c satisfy

$$5^a = 2^b = 10^{-c}, \qquad abc \neq 0.$$

What is the value of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$?

A -1

B 0

C 1

D $\frac{1}{2}$

E 2

Question 15

You may use the following result: for any triangle with area A, perimeter p and inradius r, where the inradius is the radius of the circle which touches all three sides of the triangle,

$$A = \frac{rp}{2}.$$

A triangle has integer side lengths which form a non-constant arithmetic sequence. Its inradius is 3 and its area is 54.

What is the length of its shortest side?

A 7

B 8

C 9

D 10

E 11

Question 16

The function

$$f(x) = \frac{x}{x^2 + x + 1}$$

is defined for all real x. Find the difference between its maximum and minimum values.

A $\frac{4}{3}$

B $\frac{2}{3}$

C $\frac{5}{4}$

D $\frac{3}{2}$

E 3

Question 17

Positive real numbers a, b and c satisfy $a + b + c = 6$. Find the maximum value of a^2bc.

You may find the following inequality for positive real numbers x and y useful:

$$xy \leq \left(\frac{x+y}{2}\right)^2,$$

with equality if and only if $x = y$.

A 20

B 24

C 36

D $\dfrac{81}{4}$

E 18

Question 18

Find the total length of the intervals for which

$$(2\sin x - 1)\cos\left(2x - \frac{\pi}{6}\right)\tan x \leq 0$$

for $0 < x < \pi$, with $x \neq \frac{\pi}{2}$.

A $\dfrac{5\pi}{6}$

B $\dfrac{\pi}{3}$

C $\dfrac{\pi}{4}$

D $\dfrac{5\pi}{12}$

E $\dfrac{\pi}{6}$

F $\dfrac{7\pi}{12}$

Question 19

Given that $x^3 + ax + 120 = 0$ has exactly two positive integer roots, what is the value of a?

The equation may or may not have other roots.

A -20

B -42

C -21

D -55

E -49

Question 20

You may use the following fact.

If f and g are strictly increasing functions defined for $x \geq 0$, and satisfy $f(g(x)) = x$ for all $x \geq 0$, then the graphs of f and g are reflections of each other in the line $y = x$.

Given that

$$\int_0^4 \left(2^{\sqrt{x}} - 1\right) \, dx = k \,,$$

find

$$\int_0^3 \left(\log_2(4 - x)\right)^2 \, dx$$

in terms of k.

A $2k$

B 3

C $6 - k$

D $6 + k$

E $12 - k$

F k

G $9 - k$