JZ Mock Set D Paper 2

Recommended time: 2 hours

Calculators: not permitted

Format: 20 multiple-choice questions

This is a TMUA-style mock paper modelled on the Test of Mathematics for University Admission. The TMUA is used in admissions for mathematics, economics, computer science, and engineering courses at universities including Cambridge, Oxford, Imperial College London, UCL, LSE, Warwick, and Durham.

Paper Guidance

- These papers contain only hard, very hard, and occasionally *unreasonably hard* TMUA-style questions :). I have deliberately left out the routine and easier material, so please do not be discouraged if you find a paper difficult or need longer than the recommended time. That is exactly how it is meant to feel. Take your time, enjoy the challenge, and remember that struggling with difficult problems is one of the best ways to get better.
- As a rough and deliberately conservative guide, a student who scores above 27 out of 40 across one full set, meaning both papers in four hours, is very likely already working at around the 7.0+ level on the current TMUA scale. However, this is only intended as a guide, not something to worry about too much. The main purpose of these papers is improvement rather than measurement: meet difficult problems, struggle with them, learn from them, and focus on getting better each time.

Best of luck, and enjoy the challenge!

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Question 1

How many distinct real roots does the equation $x^4 - 4x^3 + 4x^2 - 1 = 0$ have?

A 0

B 1

C 2

D 3

E 4

Question 2

Real numbers w, x, y, z all satisfy $w, x, y, z > 1$ and

$$\log_w x = x, \qquad \log_x y = y, \qquad \log_y z = z.$$

Which of the following is equal to $\log_w z$?

A xyz

B $\dfrac{1}{xyz}$

C $\dfrac{1}{wxyz}$

D $\dfrac{1}{x+y+z}$

E $\dfrac{1}{xy}$

F $\dfrac{1}{yz}$

G $x+y+z$

H $wxyz$

Question 3

Given $\tan\theta = 2,\ 90^\circ < \theta < 360^\circ$, find the value of $\sin\theta\cos\theta$.

A $\dfrac{2}{5}$

B $-\dfrac{2}{5}$

C $-\dfrac{5}{3}$

D $\dfrac{3}{5}$

E $-\dfrac{1}{5}$

Question 4

Five riders, Archimedes, Boole, Cantor, Descartes and Euler, are competing in a race. Whenever a rider changes position, the other riders keep their relative order.

Early in the race, Euler is ahead of Descartes, and Cantor is also ahead of Descartes. After a tight corner, Cantor drops back exactly one place. Later, Archimedes drops to last place. On the final lap, Descartes drops back exactly three places, and there are no further changes.

What is the final order of the riders, from first to fifth?

A Euler, Cantor, Boole, Archimedes, Descartes.

B Euler, Boole, Cantor, Archimedes, Descartes.

C Cantor, Euler, Boole, Archimedes, Descartes.

D Euler, Cantor, Archimedes, Boole, Descartes.

E Euler, Descartes, Cantor, Boole, Archimedes.

F Cantor, Boole, Euler, Archimedes, Descartes.

Question 5

Triangle ABC is right-angled at C. The perpendicular from C to the hypotenuse AB meets AB at D.

The perimeters of triangles ACD and BCD are 18 and 24 respectively. Find the area of triangle ABC.

A $\dfrac{65}{2}$

B 30

C $\dfrac{149}{4}$

D 32

E $\dfrac{75}{2}$

F 40

Question 6

Which of the following is a **sufficient** condition on the real constant k for the equation

$$(x+k)^{1/2}+(x-k)^{1/4}=k$$

to have at least one real solution for x?

A $k=0$ or $k>3$

B $k=1$ or $k>3$

C $k=0$ or $1<k<3$

D $k\geq 1$

E $0\leq k\leq 3$

Question 7

Which of the following is a **necessary but not sufficient** condition for

$$x^4 - 4x^2 + c = 0$$

to have exactly 4 distinct real roots?

A $-2 < c < 3$

B $-1 < c < 5$

C $1 < c < 6$

D $-3 < c < 2$

E $2 < c < 7$

Question 8

A TMUA question asks candidates to identify which one of the conditions below is a **sufficient but not necessary** condition for some statement P, concerning a real number x, to hold.

Friedrich Gauss, sitting the exam, noticed to his delight that the examiner has designed the question poorly, and the correct option can be deduced without knowing or examining P in the slightest!

Which is the correct option?

A $|x-2|<2$

B $x^2-5x+6<0$

C $\log(x-2)$ is defined

D $|x-2|<1$

E x can be any real number

Question 9

The real numbers x, y and z are such that exactly two of the following five statements are true.

Which two statements are true?

I $\quad x \leq y + z$

II $\quad x \leq |y| + |z|$

III $\quad y + z < |x|$

IV $\quad x + y + z > 0$

V $\quad x + y + z \leq 0$

A Statements I and II.

B Statements I and IV.

C Statements I and V.

D Statements III and IV.

E Statements II and III.

F Statements II and IV.

G Statements II and V.

H Statements III and V.

Question 10

Let a be a real number with $a > 1$, and let f be a function such that

$$f(0) = 1, \qquad f(2) > a^2, \qquad f(-1) < a^{-1}.$$

Consider the following four statements.

I. $f(x) = b^x$ for some real number b with $b > a$.

II. $f(x) = b^x$ for some real number b with $0 < b < a$.

III. $f(x) = a^{kx}$ for some real number k.

IV. $f(x) = b^{x^2}$ for some positive real number b.

Which of the statements **might** be true?

A Statement I only.

B Statement III only.

C Statements I and III only.

D Statements I, II and III only.

E Statements I, III and IV only.

F Statements II and IV only.

G Statements II, III and IV only.

H All four statements.

Question 11

In triangle ABC, $AB = t$, $BC = 10$ and $\angle BAC = 45°$, where $t > 0$.

For which values of t is the length of AC uniquely determined?

A $0 < t \leq 10$

B $0 < t < 10\sqrt{2}$

C $0 < t \leq 10$ or $t = 10\sqrt{2}$

D $t = 10\sqrt{2}$

E $t > 10$

F $0 < t \leq 10\sqrt{2}$

Question 12

Let f be a function with domain $\{1, 2, 3, 4\}$ and range contained in $\{1, 2, 3, 4\}$. For how many such functions is

$$f(f(x)) = x$$

for every x in the domain of f?

A 1

B 4

C 7

D 9

E 10

F 15

Question 13

In how many ways can 8 identical balls be placed into 3 distinct boxes so that no box contains more than 4 balls?

A 21

B 30

C 45

D 10

E 15

F 24

Question 14

For a real number x, let $\{x\}$ denote the fractional part of x. For example, $\{1.2\} = 0.2$ and $\{-1.5\} = 0.5$.

A sequence (a_n) is defined by $a_1 = \frac{1}{2}$ and

$$a_{n+1} = a_n \int_0^1 |4\{nx\} - 2| \, dx$$

for each positive integer n.

What is the value of

$$\sum_{n=1}^{\infty} a_n?$$

A 1

B 2

C 4

D 12

E $\dfrac{9}{2}$

F ∞

Question 15

A sequence is defined by $a_1 = a_2 = 1$ and $a_{n+2} = a_{n+1} + a_n$ for every positive integer n. How many of the terms $a_1, a_2, \ldots, a_{100}$ are divisible by 4?

A 8

B 16

C 12

D 5

E 15

F 20

G 24

Question 16

You are an assistant at JZ Maths, tasked with helping to set a TMUA-style multiple-choice question.

The question contains five statements, labelled $(1), (2), (3), (4), (5)$, and the student must determine exactly which statements are true. You, as the question setter, know that exactly two of the five statements are true, but this information will not be given to the student.

Each answer option is a possible set of true statements. For example, the option $\{1, 2, 5\}$ means that statements $(1), (2), (5)$ only are true, while the option $\{\}$ means that none of the statements are true.

Under Policy 1 of JZ Maths, the question must be made **as difficult as possible**: even a competent student who can correctly check every statement must determine the truth value of all five statements in order to guarantee identifying the correct option. In particular, checking any four statements must not be sufficient to determine the correct option.

Under Policy 2 of JZ Maths, the question must contain **as few answer options** as possible, so that it **appears easier** than it really is.

How many answer options should you set in order to comply with both policies? Failure to comply will result in your immediate dismissal.

Disclaimer. Unrelated to this question, please be aware that no such policies officially exist at JZ Maths of course! There may be a good reason for this, as you will learn later.

A 4

B 5

C 6

D 7

E 8

F 9

G 10

Question 17

Let f be a function defined on the real numbers.

When the graph of $y = f(x)$ is reflected in the y-axis and then stretched in the x-direction by a factor of $\frac{1}{2}$, the result is the graph of $y = g(x)$.

When the graph of $y = f(x)$ is stretched in the x-direction by a factor of $\frac{1}{2}$ and then translated by 1 unit in the negative x-direction, the result is the graph of $y = h(x)$.

Which of the following is a **sufficient but not necessary condition** for $g(x)$ and $h(x)$ to be identical, that is $g(x) = h(x)$ for all real values of x?

A $f(0) = f(2)$

B

$$f(x) = f(2 - x) \text{ for all } x$$

C

$$f(x + 1) = f(1 - x) \text{ for all } x$$

D

$$f(x) = \sum_{k=1}^{10} \cos(\pi k x)$$

E

$$f(x) = f(1 - x) \text{ for all } x$$

F

$$f(x) = \cos(\pi x) + \sin(\pi x)$$

Question 18

Find all values of k such that the equation

$$\int_0^y (x + 1 - k - |x - 1| + |x - k|)\ dx = 0$$

has exactly three distinct solutions for y, where $k > 1$ and $y \geq 0$.

A $k > 2$

B $k > 3$

C $k > 4$

D $k > 5$

E $3 < k < 4$

F $2 < k < 4$

G no such values of k exist

Question 19

Let f be a function defined on the real numbers. Consider the following seven statements, each intended to hold for all real values of a:

(1) $f(a) > 0$ if $a > 0$.

(2) $a > 0$ is necessary for $f(a) > 0$.

(3) $a \leq 0$ only if $f(a) \leq 0$.

(4) If $f(a) \leq 0$, then $a \leq 0$.

(5) $f(a) > 0$ whenever $a \leq 0$.

(6) $a > 0$ is sufficient for $f(a) > 0$.

(7) $f(a) \leq 0$ is sufficient and necessary for $a > 0$.

Given that **exactly one** of these seven statements is true for the particular function f in question, which one is it?

A Statement (1)

B Statement (2)

C Statement (3)

D Statement (4)

E Statement (5)

F Statement (6)

G Statement (7)

Question 20

Let

$$y = ax^7 + bx^6 + cx^5 + dx^4 + ex^3 + fx^2 + gx + h, \qquad a \neq 0,$$

where a, b, c, d, e, f, g, h are real numbers. Call a point on the graph an **inflection with zero gradient** if $\frac{dy}{dx} = 0$ there but $\frac{dy}{dx}$ does not change sign as x passes through it.

Which one of the following triples (number of local minima, number of local maxima, number of inflections with zero gradient) **is possible** for some choice of the real coefficients?

You may find it useful to recall that $\frac{dy}{dx}$ changes sign at a local maximum or minimum, but does not change sign at a stationary point of inflection.

A (0, 0, 4)

B (2, 2, 2)

C (2, 3, 0)

D (3, 3, 1)

E (1, 1, 3)

F (2, 1, 1)

G (2, 2, 1)