

JZ Mock Set C Paper 2

Time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

Average difficulty: 7.3

This is a TMUA-style mock paper modelled on the Test of Mathematics for University Admission. The TMUA is used in admissions for mathematics, economics, computer science, and engineering courses at universities including Cambridge, Oxford, Imperial College London, UCL, LSE, Warwick, and Durham.

Spotted an error? Please email jzmaths@hotmail.com.

Question 1

Let a, b, c be real numbers with $a \geq b$. Which of the following statements **must** be true?

1. $a^3 \geq b^3$
2. $a^2 \geq b^2$
3. $ac \geq bc$
4. $ac^2 \geq bc^2$

- A** none of them
- B** 1 only
- C** 4 only
- D** 1 and 2 only
- E** 1 and 3 only
- F** 1 and 4 only
- G** 2 and 4 only
- H** 1, 2 and 4 only
- I** 1, 3 and 4 only
- J** 1, 2, 3 and 4

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Question 2

What is the value, in radians, of the second-largest angle x in the range $0 \leq x \leq 2\pi$ that satisfies the equation

$$\tan^2(2x) \cos^2(2x) + 5 \cos^2(2x) = 4?$$

A $\frac{\pi}{12}$

B $\frac{5\pi}{12}$

C $\frac{7\pi}{6}$

D $\frac{13\pi}{12}$

E $\frac{19\pi}{12}$

F $\frac{23\pi}{12}$

G $\frac{25\pi}{12}$

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Question 3

Six sealed black bags are labelled P , Q , R , S , T and U . Each bag contains the same number of balls. This number is a non-zero whole number from 1 to 10 inclusive.

Each bag has a label with a statement written on it. The statement may be true or false.

Bag P has a label which says: "The number of balls in this bag is a multiple of 3."

Bag Q has a label which says: "The number of balls in this bag is even."

Bag R has a label which says: "The number of balls in this bag is prime."

Bag S has a label which says: "The number of balls in this bag is at most 4."

Bag T has a label which says: "The number of balls in this bag is at least 7."

Bag U has a label which says: "The number of balls in this bag is a perfect square."

Given that exactly one of the six statements is true, which bag has the true statement on its label?

A A: P

B B: Q

C C: R

D D: S

E E: T

F F: U

Question 4

Let $x \neq 0$ and

$$f(x) = \frac{3}{7}x^{7/3} + \frac{3}{4}x^{4/3} - 6x^{1/3}.$$

A student attempts to find the values of x for which the curve is increasing.

Their working is shown below.

I. $f'(x) = x^{4/3} + x^{1/3} - 2x^{-2/3}$

II. $f'(x) = x^{-2/3}(x^2 + x - 2)$

III. $x^{-2/3}(x^2 + x - 2) \geq 0$

IV. $x^2 + x - 2 \geq 0$

V. $(x - 1)(x + 2) \geq 0$

VI. $x \leq -2$ or $x \geq 1$

Which of the following is true?

- A** The first error is in step **I**.
- B** The first error is in step **II**.
- C** The first error is in step **III**.
- D** The first error is in step **IV**.
- E** The first error is in step **V**.
- F** The first error is in step **VI**.
- G** The student's answer is correct.

Question 5

A polynomial $p(x)$ satisfies $p(2) = 7$ and $p(-3) = -8$.

Which of the following can be deduced, where $q(x)$ denotes some polynomial?

A $p(x) = (x - 2)(x + 3)q(x) + 3x + 1$

B $p(x) = (x - 2)(x + 3)q(x) - 3x + 1$

C $p(x) = (x - 2)(x + 3)q(x) + 3x - 1$

D $p(x) = (x + 2)(x - 3)q(x) + 3x + 1$

E $p(x) = (x - 2)(x + 3)q(x) - x + 5$

F $p(x) = (x - 2)(x + 3)q(x) + x - 5$

G $p(x) = (x + 2)(x - 3)q(x) - 3x - 1$

H $p(x) = (x - 2)(x + 3)q(x) + \frac{15}{2}x - \frac{1}{2}$

Question 6

The curve C is defined by

$$(x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2) = 63.$$

How many points on C have both coordinates integers?

- A 0
- B 1
- C 2
- D 3
- E 4
- F 6
- G 8
- H infinitely many

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Question 7

Let $ABCD$ be a convex quadrilateral, with its vertices labelled in order. A quadrilateral is called a rhombus if all four of its sides are equal in length.

Which of the following statements, taken individually, are **sufficient** to guarantee that $ABCD$ is a rhombus?

1. $AB = AD$, and the diagonals AC and BD intersect at right angles.
2. $AB = CD$, and the diagonals AC and BD divide $ABCD$ into four similar triangles.
3. The diagonals AC and BD divide $ABCD$ into four similar triangles.
4. The diagonal AC divides $ABCD$ into two triangles, and the diagonal BD divides $ABCD$ into two triangles, with all four of these triangles having the same area.
5. $AB = BC = CD$, and the triangles ABC and BCD are congruent.

- A Only 1.
- B Only 2.
- C Only 3.
- D Only 4.
- E Only 5.
- F Only 1 and 2.
- G Only 2 and 3.
- H Only 3 and 4.
- I Only 5 and 1.
- J None of them.

Question 8

Let x and y be non-zero real numbers with $x \neq y$.

Which one of the following statements is both **necessary and sufficient** for $x < y$?

A $y^{-1/3} < x^{-1/3}$

B $y^{1/5} < x^{1/5}$

C $x^{2/3} < y^{2/3}$

D $y^{2/3} < x^{2/3}$

E $|x|^{1/3} < |y|^{1/3}$

F $x^{-1/3} < y^{-1/3}$

G $x^{1/5} < y^{1/5}$

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Question 9

Consider the following statements about the polynomial $p(x)$, where $a < b$:

(I) $p(a) < p(b)$;

(II) $p'(x) \geq 0$ for all $x \in [a, b]$;

(III) $\int_a^b p(x) dx \geq 0$;

(IV) $p(a) + p(b) \geq 2p\left(\frac{a+b}{2}\right)$.

Which of these statements is a **necessary** condition for $p(x)$ to be increasing for $a \leq x \leq b$? (Here "increasing" means $x_1 \leq x_2 \Rightarrow p(x_1) \leq p(x_2)$.)

A none of them

B I only

C II only

D III only

E IV only

F I and II only

G I and III only

H I and IV only

I II and III only

J II and IV only

K III and IV only

L I, II and III only

M I, II and IV only

N I, III and IV only

O II, III and IV only

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Question 10

A lattice path starts at the origin $(0, 0)$ and proceeds in steps of unit length along the integer lattice. The successive leg lengths are $2, 2, 4, 4, 6, 6, 8, 8, 10, 10, \dots$ (each positive even integer appearing twice in succession), and the directions cycle in the order **Right, Up, Left, Down, Right, Up, Left, Down, ...**. Thus the first eight legs of the path are R2, U2, L4, D4, R6, U6, L8, D8, and the path continues indefinitely. Which one of the following lattice points is **not** visited by the path? (Note that a visited lattice point need not be a turning point, it can be any point on the continuous path.)

- A $(99, 100)$
- B $(-99, 100)$
- C $(100, 99)$
- D $(-100, 99)$
- E $(99, -100)$
- F $(100, -99)$
- G $(-99, -100)$
- H $(-100, -99)$

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Question 11

$f(x)$ is a polynomial function defined for all real x .

Statement P: $f'(5) = 0$ and $f''(5) \geq 0$.

Statement Q: f has a local minimum at $x = 5$.

Which one of the following is true?

- A P is neither necessary nor sufficient for Q.
- B P is necessary but not sufficient for Q.
- C P is sufficient but not necessary for Q.
- D P is necessary and sufficient for Q.
- E P is sufficient for Q, and Q is sufficient for P, but they are not equivalent.
- F Whether P is necessary or sufficient for Q depends on the degree of f .

Question 12

In a TMUA-style question, students are asked to determine which, if any, of the five statements 1, 2, 3, 4, 5 are true. They must then identify the correct option from the list provided in the question.

Assume that a **competent student** can always correctly determine the truth value of any statement they choose to check.

A teacher is designing such a question and wants to choose the option set that **maximises the smallest number of checks** a lucky student might need to determine the correct option. Equivalently, the teacher wants to maximise the fewest checks after which it is *possible* for the student to know the correct option.

Which of the following four candidate option sets best achieves the teacher's aim?

Option Set I

- A: only statement 1 is true
- B: only statements 1, 2, 3 are true
- C: only statements 1, 2, 4 are true
- D: only statements 1, 4, 5 are true
- E: only statements 1, 5 are true

Option Set II

- A: only statement 1 is true
- B: only statement 2 is true
- C: only statement 3 is true
- D: only statement 4 is true
- E: only statement 5 is true

Option Set III

- A: only statements 1, 2 are true
- B: only statements 2, 3 are true
- C: only statements 3, 4 are true
- D: only statements 4, 5 are true
- E: no statements are true

Option Set IV

- A: only statements 1, 2 are true
- B: only statements 2, 3 are true
- C: only statements 3, 4 are true
- D: only statements 4, 5 are true
- E: only statements 1, 5 are true

A Option Set I

B Option Set II

C Option Set III

D Option Set IV

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Question 13

For which of the following is it a **necessary but not sufficient** condition that

$$2x^3 - 3px^2 + p = 0$$

has exactly one real root?

A $-1 < p < 1$

B $-1 \leq p \leq 1$

C $0 < p < 1$

D $p = 0$

E $p \leq -1$

F $p \geq 1$

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Question 14

Let $f(x) = x^3 - px^2 - p^2x + k$ where k is a real constant. Suppose that p and q are real numbers satisfying $-3 \leq p \leq 3$ and $-3 \leq q \leq 3$.

The function f has non-negative gradient at $x = q$.

Find the area of the region of all possible points (p, q) in the p - q plane.

A 20

B 21

C 24

D 30

E 36

F 42

G 56

H 60

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Question 15

Which one of the following numbers is largest in value?

A $\log_2 7$

B $\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k$

C $\frac{1}{\sqrt{3} - \sqrt{2}}$

D $4^{4/5}$

E $\sqrt{6 + 2\sqrt{5}}$

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Question 16

Three transformations of the plane are defined as follows. R is the reflection in the y -axis. T is the translation by 4 units in the positive x -direction. S is the horizontal stretch with scale factor 2 (parallel to the x -axis, fixing the y -axis).

Let $y = f(x)$ be a function defined on the real numbers.

When the graph of $y = f(x)$ is transformed by applying R , then T , then S (in that order), the result is the graph of $y = g(x)$.

When the graph of the same function $y = f(x)$ is transformed by applying S , then T , then R (in that order), the result is the graph of $y = h(x)$.

Which one of the following conditions on $y = f(x)$ is **necessary and sufficient** for the functions $g(x)$ and $h(x)$ to be identical?

- A $f(x) = f(x + 3)$ for all x
- B $f(x) = f(x + 4)$ for all x
- C $f(x) = f(x + 6)$ for all x
- D $f(x) = f(x + 12)$ for all x
- E $f(x) = f(-x)$ for all x
- F $f(x) = f(6 - x)$ for all x
- G $f(x) = f(8 - x)$ for all x

Question 17

Let $a, b, c > 0$. Consider the simultaneous equations:

$$\log_b a = 2, \quad \log_a c = 3, \quad \log_b(c^2 - 5c + 5) = 6.$$

Which one of the following statements about the values of a that satisfy all three equations is correct?

- A** The equations specify a uniquely.
- B** The equations are satisfied by exactly two values of a .
- C** The equations are satisfied by infinitely many values of a .
- D** The equations are contradictory; no value of a satisfies them.
- E** The equations are satisfied by exactly three values of a .

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Question 18

A student makes the following **claim**:

If f is a real-valued function such that, for some constant a , $\frac{f(x) + f(-x)}{2} = a$ for **all real values** of x , then there exists a constant k such that $f(x) = kx + a$ for **all real values** of x .

Examine their **claim** above, and determine which of the following is true?

- A The claim is true.
- B $f(x) = 2x + 1$ is a counterexample to the claim.
- C $f(x) = \sin x$ is a counterexample to the claim.
- D $f(x) = \tan x$ is a counterexample to the claim.
- E $f(x) = \cos x$ is a counterexample to the claim.
- F $f(x) = -x$ is a counterexample to the claim.

Question 19

Given that p is an integer, find the minimum value of p such that the equation

$$\sqrt{\frac{1 + \sin x}{1 - \sin 2x}} = \sqrt{\frac{1 + \sin 2x}{1 - \sin x}}$$

has exactly 16 solutions in the interval $0^\circ \leq x < p^\circ$.

A 540

B 541

C 720

D 721

E 900

F 901

G 1080

H 1081

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Question 20

An infinite sequence of integers u_1, u_2, u_3, \dots satisfies $u_1 = 10$ and

$$u_{n-1} - 9 < u_n < u_{n-1} - 2 \quad \text{for } n > 1.$$

It is given that $u_k = -50$ and that there exists some integer j with $1 < j < k$ such that $u_j = -10$.

How many different values can k take?

A 4

B 9

C 10

D 12

E 13

F 15

G 36

H 48

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