JZ Mock Set C Paper 1

Time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

Average difficulty: 6.825

This is a TMUA-style mock paper modelled on the Test of Mathematics for University Admission. The TMUA is used in admissions for mathematics, economics, computer science, and engineering courses at universities including Cambridge, Oxford, Imperial College London, UCL, LSE, Warwick, and Durham.

Spotted an error? Please email jzmaths@hotmail.com.

Question 1

Find

$$\int_1^4 \frac{(x-2)(x+3)}{x^2\sqrt{x}}\,dx.$$

A $-\frac{13}{2}$

B $-\frac{5}{2}$

C $-\frac{1}{2}$

D $-\frac{1}{4}$

E $\frac{1}{2}$

F $\frac{7}{2}$

G $\frac{13}{2}$

Question 2

The real numbers x, y, z satisfy

$$|x+2| \leq 5, \qquad |y-1| \leq 6, \qquad |z+3| \leq 4.$$

What is the greatest possible value of $|xyz|$?

A 21

B 120

C 147

D 245

E 343

F There is no greatest value.

Question 3

Which of the following five values is the **largest**?

A $\frac{7}{5}$

B $\frac{17}{12}$

C $\frac{10}{7}$

D $\frac{11}{8}$

E $2\cos\left(\frac{\pi}{4}\right)$

Question 4

Given the simultaneous equations

$$\log_{10} 2 + \log_{10}(y - 1) = \log_{10} x + \log_{10}(x - 2),$$

$$\log_{10}(y + 3 - 3x) = 0,$$

the value(s) of y are

A $10 \pm 3\sqrt{10}$

B $10 + 3\sqrt{10}$ only

C $10 - 3\sqrt{10}$ only

D $4 + \sqrt{10}$ only

E $10 \pm \sqrt{10}$

Question 5

Three players A, B and C take turns throwing a fair six-sided die in the order A, B, C, A, B, C, ... until one of them wins. The win conditions on a player's own throw are:

$$A \text{ wins if } A \text{ throws a } 6,$$
$$B \text{ wins if } B \text{ throws a } 5 \text{ or a } 6,$$
$$C \text{ wins if } C \text{ throws a } 4, 5 \text{ or } 6.$$

If a player throws and does not win, play passes to the next player. What is the probability that B is the first to win?

A $\frac{3}{13}$

B $\frac{5}{18}$

C $\frac{5}{13}$

D $\frac{6}{13}$

E $\frac{1}{3}$

F $\frac{5}{9}$

Question 6

The curve C has equation

$$y = x^2 + bx + 3$$

where $b \geq 0$. Let S denote the stationary point of C, and let O denote the origin.

Find the minimum value of

$$|OS|^2 + b^2$$

as b varies over $b \geq 0$.

A $\frac{11}{4}$

B $\frac{35}{4}$

C $\frac{39}{4}$

D $\frac{51}{4}$

E 9

Question 7

The two functions $F(n)$ and $G(n)$ are defined as follows for positive integers n:

$$F(n) = \int_0^{2n} \frac{|n - x|}{n} \, dx$$

$$G(n) = \sum_{r=1}^{n} F(r)$$

What is the largest positive integer n such that $G(n) < 210$?

A 15

B 16

C 19

D 20

E 21

Question 8

Given that

$$\int_0^1 \left(a + 3b\sqrt{x} + 2cx\right)\, dx = 5 \quad \text{and} \quad \int_0^1 \left(4a + 9b\sqrt{x} + 4cx\right)\, dx = 16,$$

find the value of $a + b$.

A 0

B 1

C 2

D 3

E 4

F 5

Question 9

The curve $x = (y-2)^2 - 1$ is transformed as follows:

First, it is translated vertically in the positive y-direction by 2 units.

Next, it is reflected in the line $x = 2$.

Finally, it is rotated 90° clockwise about the point $(1, 1)$.

Which of the following is the equation of the resulting curve?

A $y = x^2 - 8x + 13$

B $y = x^2 - 8x + 19$

C $y = -x^2 + 8x - 13$

D $x = y^2 - 8y + 15$

E $y = x^2 - 4x + 1$

Question 10

What is the smallest positive value of a for which the line $x = a$ is a line of symmetry of **both** of the curves

$$y = \sin\left(2x - \frac{\pi}{3}\right) \quad \text{and} \quad y = \cos\left(3x + \frac{\pi}{4}\right)?$$

A $\frac{\pi}{12}$

B $\frac{\pi}{4}$

C $\frac{5\pi}{12}$

D $\frac{7\pi}{12}$

E $\frac{11\pi}{12}$

F $\frac{23\pi}{12}$

Question 11

The circle C_1 has equation $x^2 + y^2 = 4$. The circle C_2 has equation $(x-9)^2 + y^2 = 25$. The straight line l is a common tangent to C_1 and C_2 such that both circles lie on the same side of l, and l has positive gradient. Find this gradient.

A $\frac{1}{3}$

B $2\sqrt{2}$

C $\frac{\sqrt{2}}{4}$

D $\frac{3\sqrt{2}}{8}$

E $\frac{\sqrt{2}}{2}$

F $\frac{2\sqrt{2}}{3}$

G $\frac{3\sqrt{2}}{4}$

H $\frac{7\sqrt{2}}{8}$

Question 12

Find the minimum value of the function $f(x) = 4^x + 4^{-x} - 3\left(2^x + 2^{-x}\right) + 5$.

A -3

B -1

C $\dfrac{3}{4}$

D 1

E 3

F 5

Question 13

The function f is defined for all real x by

$$f(x) = \frac{1}{4}x^4 - \frac{1}{3}mx^3 - m^2x^2 + n,$$

where m and n are real constants with $m > 0$.

Given that the equation $f(x) = 0$ has four distinct real roots, what is the range of values of n in terms of m?

A $0 < n < \frac{5m^4}{12}$

B $0 < n < \frac{8m^4}{3}$

C $-\frac{5m^4}{12} < n < 0$

D $-\frac{8m^4}{3} < n < 0$

E $-\frac{8m^4}{3} < n < \frac{5m^4}{12}$

F $\frac{5m^4}{12} < n < \frac{8m^4}{3}$

G $0 < n < \frac{m^4}{4}$

H No such n exists

Question 14

A positive integer k is chosen uniformly at random from $1, 2, 3, \ldots, 8$.

For each non-negative integer n, define

$$a_n = (2k+1)\sum_{r=0}^{n}\binom{n}{r}\left(-\frac{4-k}{4}\right)^r.$$

Let

$$S = a_0 + a_1 + a_2 + \cdots.$$

What is the probability that S converges to a finite value and $S > 8$?

A $\frac{1}{8}$

B $\frac{1}{4}$

C $\frac{3}{8}$

D $\frac{1}{2}$

E $\frac{5}{8}$

F $\frac{3}{4}$

G $\frac{7}{8}$

H 1

I 0

Question 15

The set of points satisfying

$$\sin(x^2 + y^2) = \frac{1}{2}, \qquad x^2 + y^2 \leq 12\pi,$$

is made up of finitely many separate closed curves. For each separate closed curve, consider the region enclosed by that curve. Find the sum of the areas of these regions.

A $31\pi^2$

B $45\pi^2$

C $60\pi^2$

D $66\pi^2$

E $78\pi^2$

F $144\pi^2$

Question 16

A sequence is defined by

$$u_n = 2\sum_{r=0}^{n-1} u_r \quad \text{for } n \geq 1, \qquad u_0 = 3.$$

Evaluate

$$\sum_{r=0}^{\infty}\left(\frac{1}{u_r}+\frac{1}{u_{r+1}}\right).$$

A the sum does not converge

B $\frac{7}{12}$

C $\frac{5}{6}$

D $\frac{7}{6}$

E $\frac{1}{2}$

Question 17

A triangle ABC is to be drawn with the following measurements:

$$AB = 3a, \qquad BC = \sqrt{5}\,a, \qquad \angle BAC = \theta^\circ$$

where $a > 0$ and $0 < \theta < 90$.

There are two possible triangles consistent with these measurements. Given the area of the larger triangle is 9 times that of the smaller triangle, what is the value of $\cos^2\theta$?

A $\dfrac{4}{9}$

B $\dfrac{4}{7}$

C $\dfrac{16}{27}$

D $\dfrac{2}{3}$

E $\dfrac{16}{21}$

F $\dfrac{64}{81}$

Question 18

A rectangle has its sides parallel to the coordinate axes and lies entirely within the region enclosed by $y = 2|x - 4|$ and $y = x + 6$.

What is the maximum possible area of the rectangle?

A $\frac{40}{3}$

B 16

C 16.5

D $\frac{49}{3}$

E 15

F $\frac{50}{3}$

G $\frac{35}{2}$

Question 19

The function f is defined for all real x by $f(x) = \cos\big(\pi \sin(\cos x)\big)$.

Let P be the smallest positive period of f, and let M be the minimum value attained by f. Which one of the following is correct?

A $P = \frac{\pi}{2}$ and $M = -1$

B $P = \frac{\pi}{2}$ and $M = \cos(\pi \sin 1)$

C $P = \pi$ and $M = -1$

D $P = \pi$ and $M = \cos(\pi \sin 1)$

E $P = \pi$ and $M = -\cos(\pi \sin 1)$

F $P = 2\pi$ and $M = -1$

G $P = 2\pi$ and $M = \cos(\pi \sin 1)$

H $P = 2\pi$ and $M = \cos 1$

Question 20

Given that

$$f(x) = \ln\left(\frac{1+x}{1-x}\right), \quad \text{where } -1 < x < 1,$$

find constants a and b such that

$$f\left(\frac{u+v}{1+uv}\right) = af(u) + bf(v).$$

Hence, or otherwise, determine which of the following is equal to

$$f\left(\frac{3x+x^3}{1+3x^2}\right) - f\left(\frac{2x}{1+x^2}\right).$$

A $\quad -2f(x)$

B $\quad -f(x)$

C $\quad f(x)$

D $\quad 2f(x)$

E $\quad 3f(x)$

F $\quad 0$