JZ Mock Set B Paper 1

Time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

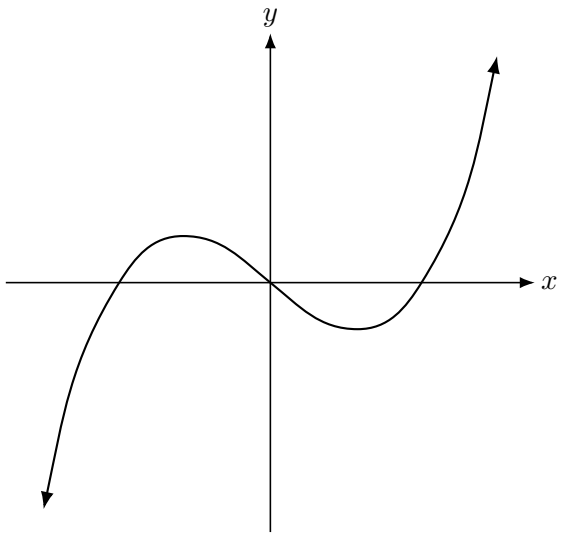
Average difficulty: 6.75

This is a TMUA-style mock paper modelled on the Test of Mathematics for University Admission. The TMUA is used in admissions for mathematics, economics, computer science, and engineering courses at universities including Cambridge, Oxford, Imperial College London, UCL, LSE, Warwick, and Durham.

Spotted an error? Please email jzmaths@hotmail.com.

Question 1

The curve $y = x^3 - 4x$ is shown below.



What is the total area enclosed between the curve, the x-axis and the lines $x = -3$ and $x = 3$?

A 0

B $\frac{9}{2}$

C 8

D $\frac{25}{2}$

E $\frac{41}{2}$

F $\frac{33}{2}$

Question 2

Evaluate

$$\sum_{n=0}^{20} \log_{\frac{1}{2}}(8^{-n}).$$

A 630

B $231 \log_{\frac{1}{2}} 8$

C $-231 \log_{\frac{1}{2}} 8$

D -693

E $-210 \log_2 8$

F 693

G $210 \log_{\frac{1}{2}} 8$

H $231 \log_2 8$

Question 3

A line is drawn normal to the curve $y = 2x^{3/2} - 4x^{1/2} - 8$ at the point on the curve where $x = 4$. This line cuts the x-axis at P and the y-axis at Q. Find the length of PQ.

A $\dfrac{2\sqrt{26}}{5}$

B $\dfrac{3\sqrt{26}}{5}$

C $\dfrac{4\sqrt{26}}{5}$

D $\dfrac{2\sqrt{23}}{5}$

E $\dfrac{3\sqrt{23}}{5}$

F $\dfrac{4\sqrt{23}}{5}$

Question 4

A circle has equation

$$x^2 + y^2 - 4x + 6y - 3 = 0.$$

The circle is first translated by 5 units in the positive x-direction, then reflected in the line $y = x$, then reflected in the x-axis, and finally enlarged by a scale factor of 3 about the origin. The equation of the final circle is

A $(x + 9)^2 + (y + 21)^2 = 144$

B $(x + 9)^2 + (y + 21)^2 = 48$

C $(x + 9)^2 + (y + 21)^2 = 16$

D $(x + 9)^2 + (y - 21)^2 = 144$

E $(x - 9)^2 + (y + 21)^2 = 144$

F $(x + 21)^2 + (y + 9)^2 = 144$

G $(x + 3)^2 + (y + 7)^2 = 144$

H $(x + 9)^2 + (y + 21)^2 = 90$

Question 5

Consider the expansion of

$$(a + bx)^n,$$

where a and b are positive real numbers and n is a positive integer.

The second term, in **ascending** powers of x, is $14x$.

The third term, in **ascending** powers of x, is $84x^2$.

The fourth term, in **descending** powers of x, is $560x^4$.

Find the value of $\frac{a}{b}$.

A $\frac{1}{4}$

B $\frac{1}{2}$

C $\frac{2}{3}$

D $\frac{5}{6}$

E 1

Question 6

Let S be the complete set of values of x satisfying both of the following:

$$x^2 > x \qquad \text{and} \qquad 1 < 17 - 4x.$$

Which one of the following is a single inequality that represents S?

A $(x-1)(x-4) < 0$

B $x^2 - 5x + 4 < 0$

C $\dfrac{3}{x-1} > x - 3$

D $x(x-1)(x-4) > 0$

E $x(x-1) < 0$

F $x^2 - x > 0$

Question 7

Let a, b and c be non-zero integers. The expression

$$\frac{6^{a+b+c} \cdot 10^{a-b-c}}{15^{a-b+c}}$$

is a positive integer **if and only if** which of the following holds?

A $a > 0$, $b > 0$ and $c < 0$

B $a > 0$, $b > 0$ and $c > 0$

C $a > 0$, $b < 0$ and $c < 0$

D $a < 0$, $b > 0$ and $c < 0$

E $a < 0$, $b < 0$ and $c > 0$

F $a < 0$, $b < 0$ and $c < 0$

G $a > 0$ and $c < 0$ (no condition on b)

H $c < 0$ only (no conditions on a or b)

Question 8

Given $-3 < x < 3$, find the total length of the intervals in which

$$\sqrt{(x-2)^2} \leq x^2 - 3x.$$

A $4 - \sqrt{3}$

B 3

C $5 - \sqrt{2}$

D $\sqrt{2} + 1$

E $\sqrt{3} + 2$

F $6 - 2\sqrt{3}$

Question 9

Evaluate the following integral.

$$\int_{-2}^{2} x^2|1-x^2|\,dx$$

A 6

B 2

C 4

D 8

E 0

Question 10

In the simplified expansion of $(4 + 15x)^{10}$, how many of the eleven terms have a coefficient that is divisible by 1000?

A 6

B 7

C 8

D 9

E 10

F 11

Question 11

The function f is continuous for all real x, and

$$\int_1^5 f(x)\,dx = A.$$

Which one of the following **must** equal to A?

A

$$\int_{-2}^{2} \left[f(x+3) + x^2\right] dx$$

B

$$\int_{-2}^{2} \left[f(x+3) + x^3\right] dx$$

C

$$\int_{-2}^{2} \left[f(x-3) + x^3\right] dx$$

D

$$\int_{-2}^{2} \left[f(x-3) + x^2\right] dx$$

E

$$\int_{-1}^{3} f(x-2)dx$$

F

$$\int_{-1}^{3} \left[f(x+2) + x^2\right] dx$$

Question 12

Three sectors of circles are similar (they share the same central angle) and their radii form an arithmetic progression. The smallest sector has arc length 6. The area of the middle sector exceeds the area of the smallest sector by 21. The area of the largest sector exceeds the area of the middle sector by 27. Find the positive difference between the perimeters of the largest and smallest sectors.

A 7

B 8

C 10.5

D 12

E 14

F 16

G 18

Question 13

Which of the following integrals has the **greatest** value?

You are not expected to calculate the exact values of any of these.

A

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^{\frac{1}{3}} x)\, dx$$

B

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos(\frac{\pi}{2} - x)|\, dx$$

C

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^2 x)\, dx$$

D

$$\int_0^{\pi} \sqrt{|\sin\left(x - \frac{\pi}{2}\right)|}\, dx$$

E

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 - \cos^2 x}\, dx$$

Question 14

Given $a_n = \sin(\frac{2\pi}{3}n) + \cos(\frac{\pi}{6}(1+4n))$ and

$$S = \sum_{n=1}^{k} a_n.$$

For how many values of positive integer k such that $1 \leq k \leq 100$, is $S = 0$.

A 0

B 33

C 34

D 66

E 67

F 99

Question 15

An arithmetic sequence (a_n) and a convergent geometric sequence (g_n) are combined to form a new sequence (T_n), where $T_n = a_n + g_n$. Given that

$$T_1 = 3, \quad T_2 = 2, \quad T_3 = \frac{3}{2}, \quad T_4 = \frac{9}{8},$$

find the sum to infinity of the geometric sequence (g_n).

A $\frac{8}{9}$

B $\frac{32}{27}$

C $\frac{4}{3}$

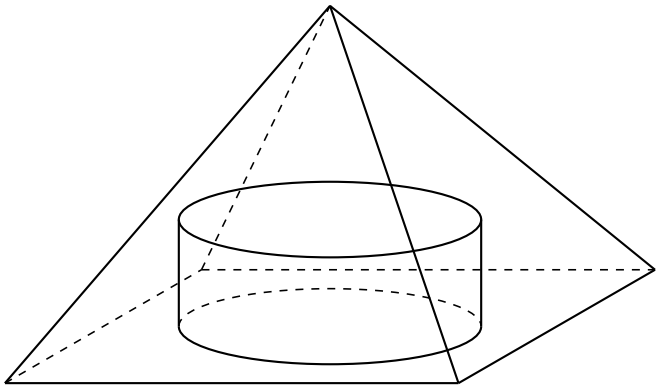
D $\frac{3}{2}$

E $\frac{27}{8}$

F 4

Question 16

A regular square-based pyramid has all edges of length L. Find the radius of the largest-volume right circular cylinder that can be enclosed inside the pyramid.



A $\frac{1}{4}L$

B $\frac{2}{5}L$

C $\frac{1}{3}L$

D $\frac{\sqrt{2}}{5}L$

E $\frac{\sqrt{2}}{4}L$

Question 17

How many solutions does the equation

$$\sqrt{1-\cos^2(2x)} = \left(x - \frac{\pi}{4}\right)\cos(2x)$$

have for x in the interval $0 < x < \pi$.

A 0

B 1

C 2

D 3

E 4

F 5

Question 18

The circles C_1 and C_2 are defined by the equations

$$C_1 : \ x^2 + y^2 - 4x - 6y + 4 = 0$$

$$C_2 : \ x^2 + y^2 - 22x - 30y + 330 = 0$$

A common tangent to C_1 and C_2 touches C_1 at P and C_2 at Q. Find the sum of the squares of all distinct possible lengths of PQ.

A 1

B 49

C 176

D 224

E 400

F 450

Question 19

Let a be a real number, and let n denote the number of points of intersection of the curves $y = |x^3 - a^3|$ and $y = a^3|x - 1|$. n **cannot** take which of the following values?

A 0

B 1

C 4

D 3

E 2

Question 20

Let $f(x) = a^{\cos x} - 2\,a^{-\cos x}$, where $a > 0$, $a \neq 1$, and x is real. The difference between the maximum value of f and the minimum value of f is $\frac{15}{2}$.

What is the sum of all possible values of a?

A $\dfrac{5}{2}$

B $\dfrac{15}{4}$

C $\dfrac{\sqrt{41}}{2}$

D $\sqrt{41}$

E $\dfrac{5+\sqrt{41}}{4}$

F $\dfrac{5+\sqrt{41}}{2}$