JZ Mock Set A Paper 1

Time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

Average difficulty: 6.975

This is a TMUA-style mock paper modelled on the Test of Mathematics for University Admission. The TMUA is used in admissions for mathematics, economics, computer science, and engineering courses at universities including Cambridge, Oxford, Imperial College London, UCL, LSE, Warwick, and Durham.

Spotted an error? Please email jzmaths@hotmail.com.

Question 1

Find the finite area enclosed between the curve $y = |x^2 - 9|$ and the line $y = 16$.

A $\frac{104}{3}$

B $\frac{142}{3}$

C $\frac{284}{3}$

D $\frac{460}{3}$

E $\frac{500}{3}$

F 60

Question 2

Find the sum of all real values of x for which there exist positive real numbers y and z satisfying the simultaneous equations

$$\log_2(x^2yz) = 3, \qquad \log_2(xz) = 1, \qquad (\log_2 y)(\log_2 z) = 2.$$

A 3

B $\frac{9}{4}$

C $\frac{9}{2}$

D 7

E 8

F 9

G $\frac{63}{4}$

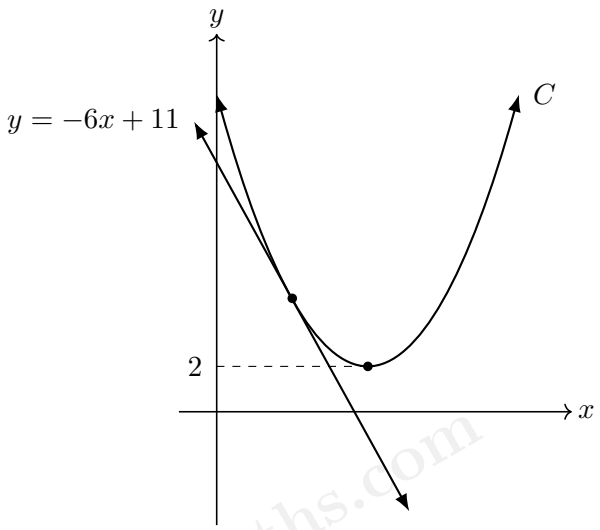
H 17

Question 3

The curve C has equation $y = px^2 + qx + r$ where p, q and r are constants with $p > 0$.

C is tangent to the line $y = -6x + 11$ at the point where $x = 1$, and the minimum value of y on C is 2.

Find the value of p.



A 1

B 2

C 3

D 4

E 5

Question 4

Let x, y and z be positive real numbers satisfying

$$18^{x+z}6^{x+y+5} = 12^{y-2}18^{5}6^{z+2}.$$

Find the value of $x + y + z$.

A 5

B 6

C 7

D 8

E 9

F 10

Question 5

A continuous function f defined on the real numbers has range $[-3, 2]$. (That is, every real value in the closed interval $[-3, 2]$ is attained by f at some real x, and no values outside this interval are attained.)

What is the difference between the maximum and the minimum of

$$(f(x))^2 + 3\,f(x) - 1$$

as x varies over the real numbers?

A $\frac{9}{4}$

B $\frac{13}{4}$

C 9

D $\frac{25}{4}$

E 10

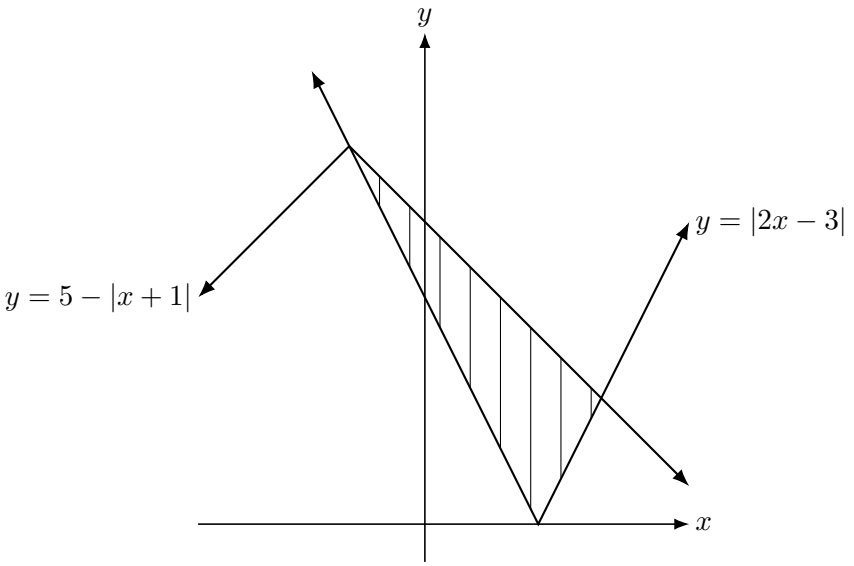
F $\frac{41}{4}$

G $\frac{45}{4}$

H $\frac{49}{4}$

Question 6

Find the area of the region enclosed between the curves $y = |2x - 3|$ and $y = 5 - |x + 1|$.



A $\frac{25}{8}$

B $\frac{50}{9}$

C $\frac{25}{6}$

D 5

E $\frac{25}{3}$

Question 7

The function

$$f(x) = \frac{1}{4}x^{4/3} + \sqrt[3]{x} + \frac{3}{\sqrt[3]{x^2}}$$

is defined for all $x \neq 0$. The complete set of values of x for which f is increasing is given by

A $-3 \leq x < 0,\ x \geq 2$

B $x \leq -3,\ 0 < x \leq 2$

C $x \geq 2$

D $-3 < x < 0,\ x > 2$

E $-3 \leq x \leq 2,\ x \neq 0$

F $-3 \leq x \leq 0,\ x \geq 2$

Question 8

In the expansion of $(a + bx)^7$, the coefficient of x^5 together with 2 times the coefficient of x^4 equals 5 times the coefficient of x^3. Given that a and b are positive integers, find the smallest possible value of $a + b$.

A 3

B 5

C 6

D 10

E 8

F 16

G 9

Question 9

For how many values of a is the equation

$$(x-a)(x^2+ax+a)=0$$

satisfied by exactly two distinct values of x?

A 0

B 1

C 2

D 3

E 4

F more than 4

Question 10

A convergent geometric progression has first term a and common ratio r. The sum to infinity of the progression is 8, and the sum to infinity of the series obtained by alternating the signs of the terms,

$$a - ar + ar^2 - ar^3 + \cdots ,$$

is 24. What is the sum to infinity of the series whose terms are the cubes of the terms of the original progression?

A 192

B 512

C 1536

D 1728

E $\frac{13824}{7}$

F The series of cubes does not converge.

Question 11

Find the set of values of k for which the equation

$$3x^4 - 8x^3 - 6x^2 + 24x + k = 0$$

has four distinct real solutions.

A $-13 < k < -8$

B $-13 < k < 19$

C $-19 < k < 13$

D $-8 < k < 19$

E $8 < k < 13$

F $k < -8$

G $k > -13$

H There are no such values of k.

Question 12

A function f satisfies

$$\int_0^2 f(x+1)\,dx = 1, \qquad \int_0^2 f(2-x)\,dx = 2, \qquad \int_0^{3/2} f(2x)\,dx = 3.$$

Find the value of $\int_1^2 f(x)\,dx$.

A -6

B -3

C 0

D 1

E 3

F 4

G 5

H 6

Question 13

The equation $5x^2 + bx + 7 = 0$ has two real roots in the ratio $2 : 5$. Find the value of b^2.

A $\frac{343}{2}$

B $\frac{343}{10}$

C $\frac{343}{50}$

D $\frac{1715}{2}$

E 1225

Question 14

The following sequence of transformations is applied, in the order listed, to the curve $y = 2x^2 - 8x + 11$: (1) translation by $\binom{-1}{4}$; (2) reflection in the line $y = 2$; (3) stretch parallel to the y-axis with scale factor 3; (4) stretch parallel to the x-axis with scale factor $\frac{1}{2}$. What is the equation of the resulting curve?

A $y = -24x^2 + 24x - 15$

B $y = -24x^2 + 24x - 27$

C $y = 24x^2 - 24x + 27$

D $y = -6x^2 + 12x - 15$

E $y = -8x^2 + 8x - 5$

F $y = -24x^2 + 72x - 63$

G $y = -8x^2 + 8x - 15$

H $y = -8x^2 + 6x - 5$

Question 15

Find the sum of the solutions of the equation

$$\sqrt{1-\sin^2 x} = 2\sin^2 x - \cos x$$

where $0 \leq x \leq 360°$.

A $180°$

B $360°$

C $540°$

D $720°$

E $900°$

Question 16

Find the maximum value of the function

$$\mathrm{f}(x) = \frac{1}{4^x + 4^{-x} - 2\left(2^x + 2^{-x}\right) + 8}.$$

A $\frac{1}{8}$

B $\frac{1}{6}$

C $\frac{1}{5}$

D $\frac{1}{4}$

E 5

F 6

Question 17

The circle C_1 has equation $(x-1)^2 + y^2 = 4$. The circle C_2 has radius 5 and centre (a, b), where (a, b) is chosen uniformly at random from the rectangle $-5 \leq a \leq 5$, $-3 \leq b \leq 3$.

What is the probability that the **circumferences** of C_1 and C_2 meet in at least one point?

A $\dfrac{3\pi}{20}$

B $\dfrac{20 - 3\pi}{20}$

C $\dfrac{60 - 49\pi}{60}$

D $\dfrac{49\pi - 9\pi}{60}$

E 1

F $\dfrac{20 - 9\pi}{20}$

Question 18

Let $0 \leq x \leq 2\pi$. Find the total length of the intervals on which all three of the following inequalities hold simultaneously

$$\sin x \geq \frac{1}{2}, \qquad \cos 2x \leq 0, \qquad \tan x \leq \sqrt{3}.$$

A $\dfrac{\pi}{12}$

B $\dfrac{\pi}{6}$

C $\dfrac{\pi}{4}$

D $\dfrac{\pi}{3}$

E $\dfrac{5\pi}{12}$

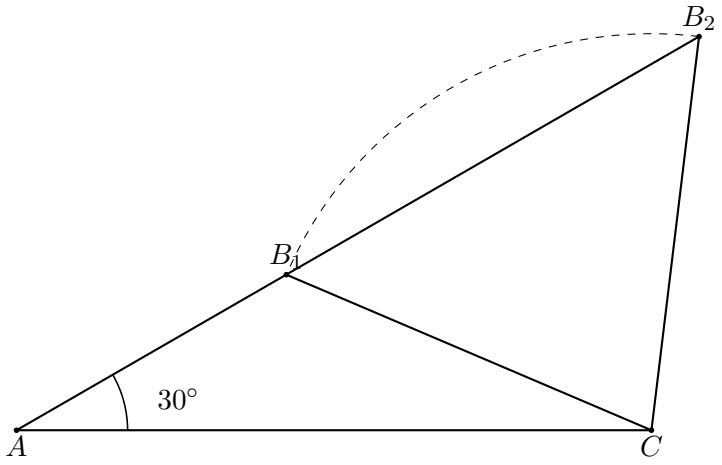
F $\dfrac{\pi}{2}$

G $\dfrac{7\pi}{12}$

H $\dfrac{2\pi}{3}$

Question 19

In a triangle ABC the angle at vertex A is $30°$. The side BC (opposite to A) has length $(x-2)(x-3)$ and the side AC (one of the sides containing A) has length $(3-x)(x-8)$. Find the complete set of values of x for which there are two non-congruent triangles with these data.



A $3 < x < 4$

B $3 < x < 5$

C $3 < x < 8$

D $4 < x < 5$

E $4 < x < 8$

F $5 < x < 8$

Question 20

A sequence of real numbers (a_n) is defined by $a_1 = 2$ and the recurrence

$$a_{n+1}(a_n - 1) = \frac{1}{2}a_n^2 - a_n + \frac{1}{2} \qquad (n \geq 1).$$

Find the value of

$$\sum_{n=1}^{\infty}(a_n + 1).$$

A $\frac{3}{2}$

B 4

C $\frac{5}{2}$

D 6

E 8

F $\frac{9}{4}$

G $\frac{5}{4}$

H ∞