

TMUA Worked Examples Worksheet 1

Question 1

A safe has a four-letter password that uses each of the letters a, b, c, d exactly once. After each attempt, the safe reports only the number of letters in the correct positions, without revealing which positions are correct. The attempt $abcd$ gave 0 correct positions, and the attempt $bcda$ also gave 0 correct positions. A player now chooses each further attempt using all previous responses. What is the smallest number N such that the player is guaranteed to identify the password after at most N further attempts?

- A 0
- B 1
- C 2
- D 3

Question 2

Let P be a statement about a real number x . Exactly one of the following conditions is necessary for P , and none of the conditions is equivalent to P .

Which of the following is a necessary but not sufficient condition for P ?

- A $-1 < x < 2$
- B $0 < x < 1$
- C $-2 < x < 3$
- D $0 < x < 2$

Question 3

Let Q be a statement about a real number x . Exactly one of the following conditions is sufficient for Q , and none of the conditions is equivalent to Q .

Which of the following is a sufficient but not necessary condition for Q ?

- A $-2 < x < 4$
- B $-1 < x < 3$
- C $0 < x < 2$
- D $0 < x < 1$

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Question 4

The real numbers x and y are such that exactly one of the following statements is true. Which is the true statement?

- A $x > 0$
- B $x + y < 0$
- C $x > y$
- D $x \leq y$

Question 5

Exactly one of the following statements is true for all real x , regarding a real-valued function g and a real number b . Which is the true statement?

- A $g(x) \geq 0$ if $x < b$.
- B $x \geq b$ is equivalent to $g(x) < 0$.
- C $g(x) < 0$ only if $x \geq b$.
- D $x < b$ and $g(x) < 0$ cannot both be true.
- E $g(x) \geq 0$ is a necessary condition for $x < b$.
- F $x \geq b$ implies $g(x) \leq 0$.

Question 6

Suppose Q is a statement that depends on the real number x . Given that exactly one of the following statements is true about Q , which is it?

- A If $|x| < 2$, then Q is true.
- B Q is false and $|x| \leq 2$ cannot both be true.
- C $1 < x < 4$ is necessary for Q to be true.
- D $|x| < 3$ only if Q is true.
- E $1 < x < 4$ if Q is true.
- F Q is false if and only if $|x| \geq 2$.

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Question 7

In triangle ABC , $\angle A = 30^\circ$, $AB = 2x^2$ and $BC = 3x$, where $x > 0$. Suppose that there are exactly two non-congruent triangles satisfying these conditions. Which of the following conditions can be deduced?

- A $1 < x < 4$
- B $0 < x < 2$
- C $2 < x < 5$
- D $\frac{1}{2} < x < \frac{5}{2}$

Question 8

You have been asked to help choose between four possible designs for a TMUA question. The question will ask students to determine which, if any, of Statements 1, 2 and 3 are true.

The statements themselves have not yet been written, so you know nothing about them except that there will be three statements to check.

As you may eventually have to answer the question yourself, you decide to choose the set of answer choices that could allow a sufficiently lucky student to determine the correct answer after checking the fewest statements. Which set of answer choices should you choose?

- A None of the statements is true.
Statement 1 only is true.
Statements 1 and 2 only are true.
All three statements are true.
- B None of the statements is true.
Statements 2 and 3 only are true.
Statements 1 and 3 only are true.
Statements 1 and 2 only are true.
- C All three statements are true.
Statement 1 only is true.
Statement 2 only is true.
Statement 3 only is true.
- D None of the statements is true.
Statement 3 only is true.
Statements 1 and 2 only are true.
All three statements are true.

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Question 9

A TMUA question setter is designing a multiple-choice question involving four statements, labelled 1, 2, 3 and 4. The setter knows that exactly two of the statements are true, but the students are not given this information.

The answer options will specify which of the four statements, if any, are true. Determined not to make the question too easy for a few lucky students, the setter wants to arrange the options so that a student must determine the truth or falsity of all four statements in order to identify the correct answer.

What is the least possible number of answer options the setter must provide?

- A 5
- B 4
- C 7
- D 6

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Question 10

The Final Easter Egg Debate

Four students, *A*, *B*, *C* and *D*, have reached the final Easter egg. This proves only that they have reached it: it does not prove that they understand it, take it seriously, or deserve an entirely imaginary biscuit.

Each student is either a **saint**, who always makes true statements, or a **liar**, who always makes false statements. No information is given about how many saints there are.

They hold the following debate. The comments in italics are merely stage directions and are not part of the evidence.

A (having read the question carefully enough to place an entirely imaginary biscuit at risk): Exactly one of us is a saint.

B (already drafting a formal complaint that the worksheet is too long): *A* and *C* are either both saints or both liars.

C (with the suspicious cheerfulness normally associated with an Easter egg question): *B* is a saint.

D (who maintains that guessing is a perfectly respectable branch of logic): *A* is a liar.

A (carefully avoiding any confusion between necessity and sufficiency): *B* is a liar.

B (now citing jzmaths.com as an independent expert witness): *D* is a saint.

C (drawing a truth table on the back of B's complaint form): *A* and *D* are either both saints or both liars.

D (requesting a second entirely imaginary biscuit before earning the first): *C* is a saint.

A (with the weary calm of someone who has survived every previous Easter egg): *C* and *D* are both liars.

Which of the following correctly identifies the students?

- A** *A* is a saint, while *B*, *C* and *D* are liars.
- B** *B* and *D* are saints, while *A* and *C* are liars.
- C** *B*, *C* and *D* are saints, while *A* is a liar.
- D** *B* and *C* are saints, while *A* and *D* are liars.