

Proof by Contradiction Worksheet 1

Question 1

Use proof by contradiction to prove that there do not exist positive integers x and y satisfying

$$4x^2 - y^2 = 9.$$

Question 2

Eva, Hugo, Maya and Zane each live on a different floor of a four-storey building. The floors are numbered 1, 2, 3 and 4, with floor 1 the lowest. There are also other people living in the building. There are 22 people living above Eva, 9 people living below Hugo, 18 people living below Maya, and 17 people living above Zane.

Use proof by contradiction to prove that Hugo cannot live on floor 2.

Question 3

Use proof by contradiction to prove that there is no smallest positive rational number.

Question 4

Use proof by contradiction to prove that, among any 13 people, at least two were born in the same month.

Proof by Contradiction Worksheet 1

Question 5

Use proof by contradiction to prove that there do not exist integers a , b and k satisfying

$$a^2 + b^2 = 4k + 3.$$

Question 6

Let n be an integer. Use proof by contradiction to prove:

If n^2 is divisible by 3, then n is divisible by 3.

Question 7

Let a and b be integers. Use proof by contradiction to prove:

If $a + b$ is odd, then exactly one of a and b is odd.

Question 8

Let x and y be real numbers. Use proof by contradiction to prove:

If $x + y > 12$, then $x > 6$ or $y > 6$.

Proof by Contradiction Worksheet 1

Question 9

Let r and s be real numbers. Use proof by contradiction to prove:

If r is rational and s is irrational, then $r + s$ is irrational.

Question 10

A triangle has side lengths a , b and c , where the angle opposite the side of length c is C . Use proof by contradiction to prove:

If $c^2 > a^2 + b^2$, then C is obtuse.

Question 11

A class contains at least two students. Use proof by contradiction to prove that it is impossible for every student in the class to have mutually complained about the Easter egg question with a different number of classmates.