Negation and Quantifiers Worksheet 3 Solutions

Question 1

Let $P(a, b, c)$ mean $c > a \Rightarrow c \geq b$. The statement translates as

$$\forall a \in S\ \exists b \in S\ \forall c \in S,\ P(a, b, c).$$

Its negation is

$$\exists a \in S\ \forall b \in S\ \exists c \in S,\ \neg P(a, b, c).$$

Since $\neg(c > a \Rightarrow c \geq b)$ means $c > a$ and $c < b$, the worded negation is option A.
The answer is **A**.

Question 2

Let $P(\epsilon, \delta, x)$ mean $|x| < \delta \Rightarrow |f(x) - f(0)| < \epsilon$. The statement translates as

$$\forall \epsilon > 0\ \exists \delta > 0\ \forall x \in \mathbb{R},\ P(\epsilon, \delta, x).$$

Its negation is

$$\exists \epsilon > 0\ \forall \delta > 0\ \exists x \in \mathbb{R},\ \neg P(\epsilon, \delta, x).$$

Since $\neg(|x| < \delta \Rightarrow |f(x) - f(0)| < \epsilon)$ means $|x| < \delta$ and $|f(x) - f(0)| \geq \epsilon$, the worded negation is option C.
The answer is **C**.

Question 3

Let $P(T, v, e)$ mean v lies on e or e is red. The statement translates as

$$\forall T \in M\ \exists v \in T\ \forall e \in T,\ P(T, v, e).$$

Its negation is

$$\exists T \in M\ \forall v \in T\ \exists e \in T,\ \neg P(T, v, e).$$

Since $\neg$(v lies on e or e is red) means v does not lie on e and e is not red, the worded negation is option D.
The answer is **D**.

Question 4

Let $P(q, b, n)$ mean note n is silver. The statement translates as

$$\forall q\ \exists b \in B_q\ \forall n \in N_b,\ P(q, b, n).$$

Its negation is

$$\exists q\ \forall b \in B_q\ \exists n \in N_b,\ \neg P(q, b, n).$$

So the worded negation is that at least one quorl has no bell whose notes are all silver, which is option B.
The answer is **B**.

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Question 5

Let $P(u, s, v)$ mean v is adjacent to $u \Rightarrow v$ is connected to s. The statement translates as

$$\forall u \in G\ \exists s \in G\ \forall v \in G,\ P(u, s, v).$$

Its negation is

$$\exists u \in G\ \forall s \in G\ \exists v \in G,\ \neg P(u, s, v).$$

Since $\neg(v$ is adjacent to $u \Rightarrow v$ is connected to $s)$ means v is adjacent to u and v is not connected to s, the worded negation is option C.
The answer is **C**.

Question 6

Let $P(a, N, n, m)$ mean $a_m > a_n$. The statement translates as

$$\forall (a_n) \in A\ \exists N \in \mathbb{Z}^+\ \forall n > N\ \exists m > n,\ P(a, N, n, m).$$

Its negation is

$$\exists (a_n) \in A\ \forall N \in \mathbb{Z}^+\ \exists n > N\ \forall m > n,\ \neg P(a, N, n, m).$$

Since $\neg(a_m > a_n)$ means $a_m \le a_n$, the worded negation is option B.
The answer is **B**.

Question 7

Let $P(r, s, P, Q)$ mean $d(P, Q) < s$ and $h(Q) < h(P) + r$. The statement translates as

$$\forall r > 0\ \exists s > 0\ \forall P \in E\ \exists Q \in E,\ P(r, s, P, Q).$$

Its negation is

$$\exists r > 0\ \forall s > 0\ \exists P \in E\ \forall Q \in E,\ \neg P(r, s, P, Q).$$

Since $\neg(d(P, Q) < s$ and $h(Q) < h(P) + r)$ means $d(P, Q) \ge s$ or $h(Q) \ge h(P) + r$, the worded negation is option A.
The answer is **A**.

Question 8

Let $P(P, l, C, Q)$ mean marked point Q lies on both l and C. The statement translates as

$$\forall P \in D\ \exists l \in D\ \forall C \in D_P\ \exists Q \in M,\ P(P, l, C, Q).$$

Its negation is

$$\exists P \in D\ \forall l \in D\ \exists C \in D_P\ \forall Q \in M,\ \neg P(P, l, C, Q).$$

So the worded negation says that for some point P, every possible line fails because some circle through P has no marked point lying on both that line and the circle. This is option D.
The answer is **D**.

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Question 9

Let $P(i, g, c, l)$ mean guardian g can see lantern l. The statement translates as

$$\forall i\ \exists g \in G_i\ \forall c \in C_i\ \exists l \in L_c,\ P(i, g, c, l).$$

Its negation is

$$\exists i\ \forall g \in G_i\ \exists c \in C_i\ \forall l \in L_c,\ \neg P(i, g, c, l).$$

So the worded negation is that one island has no guardian who can see at least one lantern in every cave. This is option C.
The answer is **C**.

Question 10

Let $P(A, a, B, b)$ mean $a + b = 0$. The statement translates as

$$\forall A \in \mathcal{F}\ \exists a \in A\ \forall B \in \mathcal{F}\ \exists b \in B,\ P(A, a, B, b).$$

Its negation is

$$\exists A \in \mathcal{F}\ \forall a \in A\ \exists B \in \mathcal{F}\ \forall b \in B,\ \neg P(A, a, B, b).$$

Since $\neg(a + b = 0)$ means $a + b \neq 0$, the worded negation is option B.
The answer is **B**.

Question 11

Let J be the set of questions on jzmaths.com, and let W_q be the set of weeks after first solving question q. Let $U(q, m)$ mean the student used method m to solve question q, and let $R(q, m, w)$ mean the student recalled method m from memory in week w.
The statement translates as

$$\forall q \in J\ \exists m\ \big(U(q, m) \text{ and } \forall w \in W_q,\ R(q, m, w)\big).$$

Its negation is

$$\exists q \in J\ \forall m\ \neg\big(U(q, m) \text{ and } \forall w \in W_q,\ R(q, m, w)\big).$$

So

$$\exists q \in J\ \forall m\ \big(\neg U(q, m) \text{ or } \exists w \in W_q,\ \neg R(q, m, w)\big).$$

This says that there is a question on jzmaths.com such that every method fails in at least one way: either the student did not use that method to solve the question, or there is a later week in which the student did not recall that method from memory. This is option A.
The answer is **A**.