Negation and Quantifiers Worksheet 2 Solutions

Question 1

Let $P(n, m)$ mean $m = n + 1$. The statement translates as

$$\forall n \in \mathbb{Z}\ \exists m \in \mathbb{Z},\ P(n, m).$$

Its negation is

$$\exists n \in \mathbb{Z}\ \forall m \in \mathbb{Z},\ \neg P(n, m).$$

So the worded negation is that at least one integer n has no integer m equal to $n + 1$.
The answer is **C**.

Question 2

Let $L(c, s)$ mean shelf s in cupboard c is labelled. The statement translates as

$$\exists c\ \forall s \in c,\ L(c, s).$$

Its negation is

$$\forall c\ \exists s \in c,\ \neg L(c, s).$$

So the worded negation is that for every cupboard, at least one shelf in it is not labelled.
The answer is **D**.

Question 3

Let $R(T, a)$ mean angle a of triangle T is a right angle. The statement translates as

$$\neg \exists T\ \exists a \in T,\ R(T, a).$$

Its negation is

$$\exists T\ \exists a \in T,\ R(T, a).$$

So the worded negation is that at least one triangle has at least one right angle.
The answer is **B**.

Question 4

Let $I(M, p)$ mean painting p in museum M is insured. The statement translates as

$$\forall M\ \forall p \in M,\ I(M, p).$$

Its negation is

$$\exists M\ \exists p \in M,\ \neg I(M, p).$$

So the worded negation is that at least one museum has at least one painting that is not insured.
The answer is **A**.

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Question 5

Let $I(p, m)$ mean moon m of planet p is icy. The statement translates as

$$\exists p \ \neg\exists m \in p,\ I(p, m).$$

Its negation is

$$\forall p \ \exists m \in p,\ I(p, m).$$

So the worded negation is that for every planet, at least one of its moons is icy.
The answer is **C**.

Question 6

Let $A(C, P)$ mean point P is on circle C, and let $X(P)$ mean P has x-coordinate 0. The statement translates as

$$\forall C \ \exists P,\ A(C, P) \wedge X(P).$$

Its negation is

$$\exists C \ \forall P,\ \neg(A(C, P) \wedge X(P)).$$

Equivalently, at least one circle has no point on it with x-coordinate 0.
The answer is **D**.

Question 7

Let $U(r, s)$ mean recipe r uses spice s. The statement translates as

$$\exists r \ \forall s \in B,\ U(r, s),$$

where B is the set of spices in the red box. Its negation is

$$\forall r \ \exists s \in B,\ \neg U(r, s).$$

So the worded negation is that every recipe misses at least one spice in the red box.
The answer is **A**.

Question 8

Let $G(g, w)$ mean wing w of glimwick g is glowing. The statement translates as

$$\neg\exists g \ \forall w \in g,\ G(g, w).$$

Its negation is

$$\exists g \ \forall w \in g,\ G(g, w).$$

So the worded negation is that at least one glimwick has every wing glowing.
The answer is **C**.

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Question 9

Let $P(x, y)$ mean $xy = 1$. The statement translates as

$$\forall x \in \mathbb{R}^+ \; \exists y \in \mathbb{R}^+, \; P(x, y).$$

Its negation is

$$\exists x \in \mathbb{R}^+ \; \forall y \in \mathbb{R}^+, \; \neg P(x, y).$$

So the worded negation is that at least one positive real number x has no positive real number y with $xy = 1$.
The answer is **B**.

Question 10

Let $O(c, d)$ mean digit d in door code c is odd. The statement translates as

$$\exists c \; \forall d \in c, \; O(c, d).$$

Its negation is

$$\forall c \; \exists d \in c, \; \neg O(c, d).$$

So the worded negation is that every door code has at least one digit that is not odd.
The answer is **D**.

Question 11

Let $T(s, p, g)$ mean page g in project p by student s has a title. The statement translates as

$$\forall s \; \exists p \in s \; \forall g \in p, \; T(s, p, g).$$

Its negation is

$$\exists s \; \forall p \in s \; \exists g \in p, \; \neg T(s, p, g).$$

So the worded negation is that at least one student has the property that every one of that student's projects has at least one page with no title.
The answer is **C**.

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Question 12

Let $W(c, p, f)$ mean fountain f in park p in city c is working. The statement translates as

$$\exists c\ \forall p \in c\ \exists f \in p,\ W(c, p, f).$$

Its negation is

$$\forall c\ \exists p \in c\ \forall f \in p,\ \neg W(c, p, f).$$

So the worded negation is that for every city, at least one park in that city has no working fountain.
The answer is **B**.

Question 13

Let $R(a, h, s)$ mean scroll s on shelf h of archivist a is red. The statement translates as

$$\neg\exists a\ \exists h \in a\ \forall s \in h,\ R(a, h, s).$$

Its negation is

$$\exists a\ \exists h \in a\ \forall s \in h,\ R(a, h, s).$$

So the worded negation is that at least one archivist has at least one shelf on which every scroll is red.
The answer is **D**.

Question 14

Let $P(x, y, z)$ mean $x + y < z$. The statement translates as

$$\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ \forall z \in \mathbb{R}^+,\ P(x, y, z).$$

Its negation is

$$\exists x \in \mathbb{R}\ \forall y \in \mathbb{R}\ \exists z \in \mathbb{R}^+,\ \neg P(x, y, z).$$

Since $\neg(x + y < z)$ means $x + y \geq z$, this matches option A.
The answer is **A**.

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Question 15

Let $S(b, r, p)$ mean robot b scans panel p in room r. The statement translates as

$$\exists b\ \forall r\ \exists p \in r,\ S(b, r, p).$$

Its negation is

$$\forall b\ \exists r\ \forall p \in r,\ \neg S(b, r, p).$$

So the worded negation is that every robot has at least one room in which no panel is scanned by that robot.
The answer is **C**.

Question 16

Let $S(T, v, e)$ mean side e touching vertex v in triangle T is shorter than 10 cm. The statement translates as

$$\forall T\ \exists v \in T\ \forall e \sim v,\ S(T, v, e).$$

Its negation is

$$\exists T\ \forall v \in T\ \exists e \sim v,\ \neg S(T, v, e).$$

So the worded negation is that at least one triangle has the property that every vertex touches at least one side that is not shorter than 10 cm.
The answer is **D**.

Question 17

Let $F(b, c, r)$ mean rune r in chapter c of spellbook b is forbidden. The statement translates as

$$\exists b\ \neg\exists c \in b\ \exists r \in c,\ F(b, c, r).$$

Its negation is

$$\forall b\ \exists c \in b\ \exists r \in c,\ F(b, c, r).$$

So the worded negation is that every spellbook has at least one chapter containing at least one forbidden rune.
The answer is **B**.

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Question 18

Let $R(i, b, v)$ mean bridge b on island i reaches village v. The statement translates as

$$\forall i\ \exists b \in i\ \forall v \in i,\ R(i, b, v).$$

Its negation is

$$\exists i\ \forall b \in i\ \exists v \in i,\ \neg R(i, b, v).$$

So the worded negation is that at least one island has the property that every bridge fails to reach at least one village on that island.
The answer is **A**.

Question 19

Let $G(p, n, c)$ mean coefficient c of polynomial p is greater than integer n. The statement translates as

$$\exists p\ \forall n \in \mathbb{Z}\ \exists c \in p,\ G(p, n, c).$$

Its negation is

$$\forall p\ \exists n \in \mathbb{Z}\ \forall c \in p,\ \neg G(p, n, c).$$

Since $\neg(c > n)$ means $c \leq n$, this matches option D.
The answer is **D**.

Question 20

Let $C(m, r)$ mean crater r is on moon m, and let $M(s, m)$ mean starwhale s carries a map for moon m. The statement translates as

$$\neg\exists s\ \forall m,\ (\exists r,\ C(m, r)) \Rightarrow M(s, m).$$

Its negation is

$$\exists s\ \forall m,\ (\exists r,\ C(m, r)) \Rightarrow M(s, m).$$

So the worded negation is that at least one starwhale carries a map for every moon that has at least one crater.
The answer is **C**.