Counterexamples Worksheet 1 Solutions

Question 1

A counterexample to an if-then statement must make the if part true and the then part false. A counterexample must satisfy the if part and make the then part false. The integer 8 is divisible by 4, but it is not odd. The other relevant numerical choice, 9, does not satisfy the if part. Hence option A is valid.
The answer is **A**.

Question 2

A counterexample to an if-then statement must make the if part true and the then part false. For $x = -4$, the if part is true because $(-4)^2 = 16 > 9$, but the then part is false because $-4 > 3$ is false. Thus option B is a valid counterexample.
The answer is **B**.

Question 3

A counterexample to an if-then statement must make the if part true and the then part false. An equilateral triangle has three equal sides, so it satisfies the if part. Its angles are all $60°$, so it has no right angle. Therefore option B is a valid counterexample.
The answer is **B**.

Question 4

A counterexample to an if-then statement must make the if part true and the then part false. The function $f(x) = x^2$ is even, so the if part is true. But $f(1) = 1$, not 0, so the then part is false. Thus option C is a valid counterexample.
The answer is **C**.

Question 5

A counterexample to an if-then statement must make the if part true and the then part false. Option C makes the if part true because the bus leaves before 8:00 a.m., but makes the then part false because it does not arrive before 9:00 a.m. Hence it is the valid counterexample.
The answer is **C**.

Question 6

A counterexample to an if-then statement must make the if part true and the then part false. A 2 by 5 rectangle has two pairs of parallel sides, so the if part is true. Its four sides are not all equal, so the then part is false. Therefore option C is a valid counterexample.
The answer is **C**.

Question 7

A counterexample to an if-then statement must make the if part true and the then part false. The values $-1, -2, -5$ all satisfy the if part and have negative cubes, so each is a counterexample. The value 2 does not satisfy the if part, so it is not a valid counterexample. The answer is D.
The answer is **D**.

Counterexamples Worksheet 1 Solutions

Question 8

A counterexample to an if-then statement must make the if part true and the then part false. The sequence $a_n = n - 5$ is strictly increasing, but its first term is $a_1 = -4$, which is not positive. Thus option B satisfies the if part and violates the then part.
The answer is **B**.

Question 9

A counterexample to an if-then statement must make the if part true and the then part false. In option A, the customer buys four books, so the if part is true, but spends only £16, so the then part is false. Hence option A is a valid counterexample.
The answer is **A**.

Question 10

A counterexample to an if-then statement must make the if part true and the then part false. The polynomial $x^2 + 1$ has degree 2, but it has no real roots. It therefore satisfies the if part and makes the then part false. Option B is the valid counterexample.
The answer is **B**.

Question 11

A counterexample to an if-then statement must make the if part true and the then part false. The outer if part requires n to be prime. For $n = 2$, this is true, but the positive divisor 2 is not odd, so the universal claim in the then part fails. Thus option A is valid.
The answer is **A**.

Question 12

A counterexample to an if-then statement must make the if part true and the then part false. The function in option B is quadratic and -1 is one of its real roots. Since -1 is not positive, the quantified then part fails. The other options either fail the outer if part or do not provide a root that violates the claim.
The answer is **B**.

Question 13

A counterexample to an if-then statement must make the if part true and the then part false. An equilateral triangle is isosceles because it has at least two equal sides. However, all its angles are exactly 60°, so there is no vertex angle greater than 60°. Hence option B is a valid counterexample.
The answer is **B**.

Question 14

A counterexample to an if-then statement must make the if part true and the then part false. Option C makes the if part true because the shop is open. The item costing £25 shows that the claim about every displayed item is false. Therefore option C is valid.
The answer is **C**.

Counterexamples Worksheet 1 Solutions

Question 15

A counterexample to an if-then statement must make the if part true and the then part false. For option A, 12 is a multiple of 4 greater than 4, and 3 is a factor with $2 < 3 < 12$. But 3 is not even, so the universal claim fails. In B the outer if part fails, in C the proposed m is not a factor, and in D the conclusion holds.
The answer is **A**.

Question 16

A counterexample to an if-then statement must make the if part true and the then part false. To make the existential then part false, every possible switch must fail. Option C does exactly this: the machine is certified, but each switch has at least one linked lamp that is not green. Hence no suitable switch exists.
The answer is **C**.

Question 17

A counterexample to an if-then statement must make the if part true and the then part false. The set in option B is non-empty and finite. For $a = -2$, taking $b = -2$ gives $a + b = -4 \leq 0$; for $a = -1$, taking $b = -2$ gives $a + b = -3 \leq 0$. Thus no a satisfies the required universal condition, so B is a valid counterexample.
The answer is **B**.

Question 18

A counterexample to an if-then statement must make the if part true and the then part false. An equilateral triangle is convex, but it has no diagonals at all. Therefore, at each vertex, the claim that there exists a suitable diagonal fails. Option A is a valid counterexample. The concave quadrilateral does not satisfy the if part.
The answer is **A**.

Question 19

A counterexample to an if-then statement must make the if part true and the then part false. The app in option C satisfies the if part. However, every server fails the required universal condition because each has at least one user waiting too long. Therefore no suitable server exists, so C is a valid counterexample.
The answer is **C**.

Question 20

A counterexample must satisfy the condition of the statement but fail to satisfy its conclusion. Therefore, the student must complete every Logic worksheet on jzmaths.com, but must subsequently reach at least one invalid conclusion.
The answer is **A**.
The statement is not true, of course, though I wish it were! Completing every worksheet will, however, improve a student's logic, even if it does not grant logical infallibility! :)