

Counterexamples Worksheet 1

Question 1

Consider the statement:

If an integer n is divisible by 4, then n is odd.

Which of the following is a valid counterexample?

- A $n = 8$
- B $n = 9$
- C $n = 12$ and n is odd
- D $n = 3$

Question 2

Consider the statement:

If a real number x satisfies $x^2 > 9$, then $x > 3$.

Which of the following is a valid counterexample?

- A $x = 4$
- B $x = -4$
- C $x = 2$
- D $x = 3$

Question 3

Consider the statement:

If a triangle has three equal sides, then it has a right angle.

Which of the following is a valid counterexample?

- A A right-angled isosceles triangle
- B An equilateral triangle
- C A scalene triangle with one right angle
- D A triangle with side lengths 2, 3, 4

Counterexamples Worksheet 1

Question 4

Consider the statement:

If a function f is even, then $f(1) = 0$.

Which of the following is a valid counterexample?

- A $f(x) = x$
- B $f(x) = x^3$
- C $f(x) = x^2$
- D $f(x) = x - 1$

Question 5

Consider the statement:

If a bus leaves its depot before 8:00 a.m., then it arrives at the station before 9:00 a.m.

Which of the following is a valid counterexample?

- A The bus leaves at 8:10 a.m. and arrives at 8:50 a.m.
- B The bus leaves at 7:50 a.m. and arrives at 8:45 a.m.
- C The bus leaves at 7:50 a.m. and arrives at 9:10 a.m.
- D The bus leaves at 9:00 a.m. and arrives at 9:20 a.m.

Question 6

Consider the statement:

If a quadrilateral has two pairs of parallel sides, then all four of its sides are equal.

Which of the following is a valid counterexample?

- A A square
- B A rhombus
- C A rectangle with side lengths 2 and 5
- D A kite with no parallel sides

Counterexamples Worksheet 1

Question 7

Consider the statement:

If an integer n is negative, then n^3 is positive.

Which of the following is **not** a valid counterexample?

- A $n = -1$
- B $n = -2$
- C $n = -5$
- D $n = 2$

Question 8

Consider the statement:

If a sequence is strictly increasing, then its first term is positive.

Which of the following is a valid counterexample?

- A $a_n = n$
- B $a_n = n - 5$
- C $a_n = -n$
- D $a_n = 5 - n$

Question 9

Consider the statement:

If a customer buys more than three books, then the customer spends more than £20.

Which of the following is a valid counterexample?

- A The customer buys four books costing £4 each.
- B The customer buys three books costing £9 each.
- C The customer buys five books costing £5 each.
- D The customer buys two books costing £8 each.

Counterexamples Worksheet 1

Question 10

Consider the statement:

If a polynomial has degree 2, then it has two distinct real roots.

Which of the following is a valid counterexample?

- A $x^2 - 1$
- B $x^2 + 1$
- C $x - 1$
- D $x^3 - x$

Question 11

Consider the statement:

If n is a prime number, then every positive divisor of n is odd.

Which of the following is a valid counterexample?

- A $n = 2$, using the divisor 2
- B $n = 3$, using the divisor 1
- C $n = 4$, using the divisor 2
- D $n = 5$, using the divisor 5

Question 12

Consider the statement:

If f is a quadratic polynomial, then every real root of f is positive.

Which of the following is a valid counterexample?

- A $f(x) = x^3 - x$, using the root $x = -1$
- B $f(x) = x^2 - 1$, using the root $x = -1$
- C $f(x) = x^2 + 1$, using $x = -1$
- D $f(x) = x^2 - 4x + 3$, using the root $x = 1$

Counterexamples Worksheet 1

Question 13

Consider the statement:

If a triangle is isosceles, then there exists a vertex angle greater than 60° .

Which of the following is a valid counterexample?

- A An isosceles triangle with angles $50^\circ, 50^\circ, 80^\circ$
- B An equilateral triangle
- C A scalene triangle with angles $50^\circ, 60^\circ, 70^\circ$
- D A right-angled isosceles triangle

Question 14

Consider the statement:

If a shop is open, then every item displayed in its front window costs less than £20.

Which of the following is a valid counterexample?

- A The shop is closed and a displayed coat costs £80.
- B The shop is open and every displayed item costs £15.
- C The shop is open and one displayed bag costs £25.
- D The shop is closed and every displayed item costs £10.

Question 15

Consider the statement:

If n is a multiple of 4 greater than 4, then every factor m of n satisfying $2 < m < n$ is even.

Which of the following is a valid counterexample?

- A $n = 12$, using $m = 3$
- B $n = 10$, using $m = 5$
- C $n = 12$, using $m = 5$
- D $n = 8$, using $m = 4$

Counterexamples Worksheet 1

Question 16

Consider the statement:

If a machine is certified, then there exists a switch such that every lamp linked to that switch is green.

Which of the following describes a valid counterexample?

- A A machine that is not certified and has no green lamps
- B A certified machine with one switch whose linked lamps are all green
- C A certified machine in which every switch has at least one linked lamp that is not green
- D A machine that is not certified but has a switch whose linked lamps are all green

Question 17

Consider the statement:

If S is a non-empty finite set of real numbers, then there exists a in S such that, for every b in S , $a + b > 0$.

Which of the following is a valid counterexample?

- A $S = \{1, 2\}$
- B $S = \{-2, -1\}$
- C $S = \{-1, 2\}$, using $a = 2$
- D $S = \emptyset$

Question 18

Consider the statement:

If a polygon is convex, then for every vertex V there exists a diagonal from V that is longer than every side of the polygon.

Which of the following is a valid counterexample?

- A An equilateral triangle
- B A square of side length 1
- C A regular pentagon
- D A concave quadrilateral

Counterexamples Worksheet 1

Question 19

Consider the statement:

If an app is online, then there exists a server such that every user connected to that server receives a response within one second.

Which of the following describes a valid counterexample?

- A The app is offline and every server is slow.
- B The app is online and one server responds to all its users within one second.
- C The app is online and every server has at least one connected user who waits longer than one second.
- D The app is offline but one server responds quickly.

Question 20

Consider the statement:

If a student completes every Logic worksheet on jzmaths.com, then every conclusion subsequently reached by that student is valid.

Which of the following describes a valid counterexample?

- A A student who completes every Logic worksheet and subsequently reaches at least one invalid conclusion.
- B A student who completes every Logic worksheet and subsequently reaches only valid conclusions.
- C A student who has not completed every Logic worksheet and subsequently reaches an invalid conclusion.
- D A student who has not completed every Logic worksheet and subsequently reaches only valid conclusions.